NEW AGE

Second Edition

Plane Surveying

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NEW AGE INTERNATIONAL PUBLISHERS

1

INTRODUCTION AND BASIC CONCEPTS

1.1 GENERAL

The plans or maps of areas are required for various purposes such as planning, design and execution of engineering projects, location of different geological structures, showing the geographical boundaries of districts, states and countries too. The plans or maps are prepared by the method of surveying. *Surveying* is defined as an art of making such measurements of the relative positions of points on the surface of earth that on drawing them to scale, natural and artificial features are exhibited in their correct relative horizontal and vertical positions.

1.2 PRIMARY DIVISIONS OF SURVEYING

The earth has the approximate shape of an oblate ellipsoid of revolution. The Clark (1866) ellipsoid has polar axis of 12,713,168 m and an equatorial axis of 12,756,413 m. The difference in the two axes is only about 43 km.

The shape of the earth being close to an ellipsoid, its surface has curvature. The curvature of the earth's surface is the main factor for the following divisions of surveying:

(*a*) Geodetic Surveying
(*b*) Plane Surveying.

Geodetic Surveying

The geodetic surveying is that type of surveying which takes into account the curvature of the earth's surface for achieving high precision using principles of geodesy. It generally extends over large areas.

Plane Surveying

The plane surveying is that type of surveying which does not take into account the curvature of the earth, and the surface of the earth is treated as a plane surface. If the area to be surveyed is more than 1000 km^2, the angles measured on the surface of the earth cannot be in a plane surface, but are in a curved surface, therefore, geodetic surveying must be employed to achieve precision for large areas.

1.3 CLASSIFICATION OF SURVEYING

Broadly the classification of surveying may be based on

(*a*) the function of survey, and
(*b*) instruments employed.

Classification Based on Function of Surveying

The surveys can further be classified based on the purpose for which they are conducted. They are as follows.

Control surveying is for establishing the horizontal and vertical positions of widely spaced control points using geodetic methods.

Land surveying is for determining the boundaries and areas of parcels of land. It is also known as property survey, boundary survey, or cadastral survey.

City surveys are for urban planning. These are conducted within the limits of the city. These are required for the purpose of layout of buildings, streets, sewers, etc.

Topographic surveys are for depiction of topography of a region. It includes natural features such as hills, rivers, forest, and man-made features such as towns, villages, buildings, roads, transmission lines, and canals.

Engineering surveys are undertaken specifically for engineering purposes. It is for collection of requisite data for planning, design, and execution of engineering projects such as design of roads, bridges, dams, transmission lines. There are further subdivisions as reconnaissance, preliminary surveys, and location surveys. The first is of exploratory nature, the second is to collect adequate data for design, and the third is to set out work on the ground.

Route surveys are primarily for planning, design, and execution of highways, railways, canals, pipelines, and other linear projects.

Construction surveys are those types of surveys which are required to establish points, lines, grades, and for staking out engineering works, after the plans have been prepared and the structural design has been done.

Astronomic surveys are conducted for the determination of latitudes, longitudes, azimuths, local time, etc., for various places by observing heavenly bodies such as Sun and stars.

Geological surveys are conducted to determine different strata of the earth's crust for geological studies.

Archaeological surveys have the primary objective of unearthing relics of antiquity.

Mine surveys are conducted for the exploration of mineral deposits, and to guide tunnelling and other operations associated with mining.

Satellite surveys are conducted to establish intercontinental, interdatum, and interisland geodetic ties all the world over by making observations on artificial satellites.

Military surveys are conducted for military purposes.

Classification Based on Instruments Employed

The surveying operations employ various kinds of instruments, and therefore, it is possible to classify surveys according to the principal instrument on which they are based as under.

Chain survey: This is the simplest type of surveying in which only linear measurements are made with a chain or tape, and no angular measurements are taken.

Compass survey: The horizontal angles are measured with the help of a magnetic compass. The linear measurements are also required which are taken with a chain or tape.

Plane-table survey: The map is prepared in the field itself by determining the directions of various lines making linear measurements, and plotting the details on paper using a plane table.

Levelling survey: This type of survey is used to determine the elevations and relative heights of points with the help of an instrument known as level.

Theodolite survey: Theodolite survey is primarily used in traversing and triangulation for providing controls. The horizontal and vertical angles are measured with the help of theodolite.

Tacheometric survey: A special type of theodolite known as tacheometer, is used to determine horizontal and vertical distances directly.

Photogrammetric survey: In this type of survey, the measurements are made with the help of photographs.

EDM survey: In this type of survey, the linear measurements are made with the help of EDM instruments. In trilateration, all the three sides of a triangle are measured with EDM instrument.

1.4 PRINCIPLES OF SURVEYING

The fundamental principles upon which the various methods of surveying are based, are very simple to understand. They involve

1. fixing a point in relation to points already fixed, and
2. working from the whole to the part.

Fixing a Point in Relation to Points Already Fixed

In Fig.1.1, the positions of two points *A* and *B* are already fixed. The third point *C* can be located in relation to *A* and *B*, by the following direct approaches:

(*a*) Measure the distances *AC* and *BC*, and locate *C* as the intersection point of the arcs with centres at *A* and *B*.

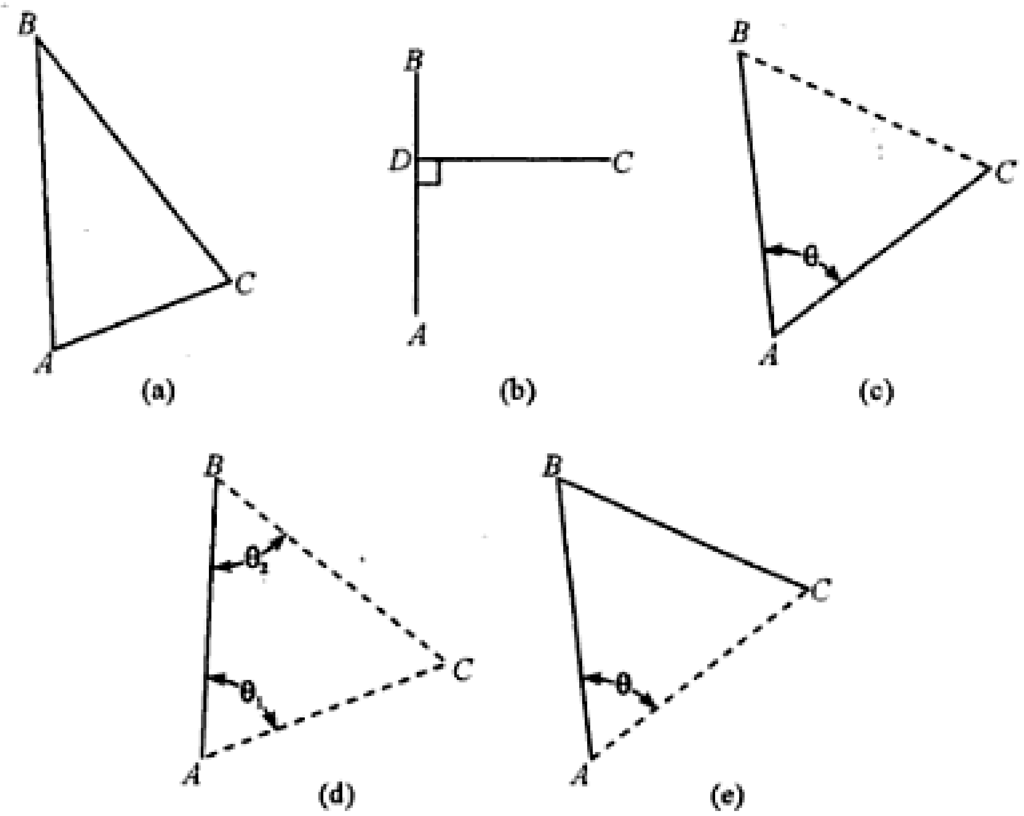


Fig. 1.1 Fixing a point in relation to points already fixed

(*b*) Measure the perpendicular distance *CD* of *C* from *AB* and the distance *AD* or *BD*, to locate *C*.

(*c*) Measure the distance *AC* and angle *BAC*, to locate *C*.

(*d*) Measure the angles *BAC* and *ABC*, to locate *C*.

(*e*) Measure the angle *BAC* and the distance *BC*, to locate *C*.

The plotting of points on drawing sheet is done by scaling the measured distances.

The above methods, specially (*b*), (*c*), and (*d*) could also be employed for measuring the relative altitudes.

For a given survey work, depending on the purpose of the survey, the degree of accuracy required, the nature and the extent of the area, and the time available, a surveyor may adopt different combinations of the above approaches and various types of available instruments for fixing horizontal and vertical locations of points.

The selection of a method best suited to a particular survey depends on the degree of skill acquired by the surveyor.

Working From the Whole to the Part

This is the ruling principle in surveying. The surveyor should first establish a sufficient number of points with high degree of precision in and around the area to be surveyed. Such points are known as *primary control points*. The gaps are then filled with a system of *secondary control points* at closer intervals with slightly less precision. Further gaps are then filled by *tertiary control points* at even closer intervals and with even less precision. For the surveys which are of ordinary nature, the tertiary control points are used to fix details on the ground. As a rule, the errors in survey details should be too small to plot, while the accuracy of the control points used for plotting the detail must be as high as possible.

The purpose of working from the whole to the part is mainly to localise the errors, i.e., not to magnify, and to control the accumulation of errors. In the reverse process of working from the part to the whole, the errors will get magnified.

The other points which must be kept in mind are:

(*a*) Choose a method of survey that is the most suitable for the purpose.

(*b*) Recording of data should be done carefully.

(*c*) Make provisions for adequate checks.

1.5 DEFINITIONS OF SOME BASIC TERMS

The following are some basic terms most commonly used in surveying:

Level surface: A level surface is the equipotential surface of the earth's gravity field. It is a curved surface and every element of which is normal to plumb line. A body of still water provides the best example of a level surface.

Level line: A line lying in a level surface is a level line. It is thus a curved line normal to the plumb at all points.

Horizontal plane: A horizontal plane is a plane which is tangential to a level surface at a particular point.

Horizontal line: A line tangent to a level surface is a horizontal line.

Horizontal angle: An angle measured between two intersecting lines in a horizontal plane, is defined as a horizontal angle.

Vertical line: A vertical line is a line perpendicular to the horizontal plane.

Vertical plane: A plane containing a vertical line is vertical plane.

Vertical angle: The angle between two intersecting lines in a vertical plane, is vertical angle.

Zenith angle: An angle between two lines in a vertical plane where one of the lines is directed towards zenith is known as zenith angle. A vertical line in the direction away from the centre of the earth and above the observer's head, is said to be directed towards zenith.

Horizontal distance: In plane surveying, distance measured along a level line is termed as horizontal distance.

Elevation: The vertical distance of a point from an assumed datum or mean sea-level is known as elevation.

Contour: A contour is an imaginary line of constant elevation on the surface of ground.

Grade or gradient: The slope of a line or rate of ascent or descent is termed as grade or gradient.

Latitude and departure: If the x-axis and y-axis in a cartesian coordinate system, are in east-west direction and north-south direction, respectively, the y-cordinate of point is its latitude and x-coordinate departure.

1.6 PHASES OF SURVEY WORK

A survey work has the following phases:

Planning, analysis and decision-making It involves the following:

(*i*) Selection of an appropriate method of surveying
(*ii*) Selection of instruments and other equipments
(*iii*) Selection and fixing of survey stations.

Care and adjustment of instruments To ensure best results, the instruments must be kept in good working condition. The surveyor must check before use that the permanent adjustments of the instruments are not disturbed.

Field work It is the process of collecting field data by making linear and angular measurements, and recording them in a systematic manner.

Office work The office work involves:

(*i*) Computation of coordinates
(*ii*) Data processing
(*iii*) Preparing plans or maps to suitable scales
(*iv*) Computation of areas and volumes.

Setting out works It involves pegging out the structure on the ground before construction starts.

1.7 SCALE

Since the areas to be plotted are rather large compared to the size of the paper on which the map of the area is to be plotted, it is not possible to draw the measured distances directly. Actually, the measured distances are drawn to some scales.

The *scale* is defined as the ratio of the distance between two points on a map to the corresponding distance on the ground. Selection of a scale depends on the size of the area to be surveyed, purpose of survey, and required precision in plotting. Maps are generally classed as *large scale* when the scale is greater than 1 cm = 10 m, as *intermediate scale* when scale is between 1 cm = 10 m and 1 cm = 100 m, and as *small scale* when the scale is less than 1 cm = 100 m. In general, the scale should be as small as possible and still represent the detail with sufficient precision. Thus, it should not be necessary to read the scale closer than 0.25 mm or so.

Table 1.1 gives the recommended scales for different type of surveys.

Table 1.1 Scales for different type of surveys

Purpose of survey	Scale	R.F.
Building sites	1 cm = 10 m or less	$\frac{1}{1000}$ or less
Town planning, reservoir survey, etc.	1 cm = 50 m to 100 m	$\frac{1}{5000}$ to $\frac{1}{10,000}$
Route surveys	1 cm = 100 m	$\frac{1}{10,000}$
Longitudinal section		
I: Horizontal scale	1 cm = 100 m to 200 m	$\frac{1}{10,000}$ to $\frac{1}{20,000}$
II: Vertical scale	1 cm = 1 m to 2 m	$\frac{1}{100}$ to $\frac{1}{200}$
Cross sections (Both horizontal and vertical scales are equal)	1 cm = 1 m to 2 m	$\frac{1}{100}$ to $\frac{1}{200}$
Location surveys	1 cm = 50 m to 200 m	$\frac{1}{5000}$ to $\frac{1}{20,000}$
Land surveys	1 cm = 5 m to 50 m	$\frac{1}{500}$ to $\frac{1}{5,000}$
Topographic surveys	1 cm = 0.25 km to 2.5 km	$\frac{1}{25,000}$ to $\frac{1}{250,000}$
Cadastral surveys	1 cm = 5 m to 0.5 km	$\frac{1}{500}$ to $\frac{1}{50,000}$
Geographical surveys	1 cm = 5 km to 150 km	$\frac{1}{5,00,000}$ to $\frac{1}{150,00,000}$

Representation of Scale

A scale can be represented numerically or graphically. If a scale is represented as 1 cm = 50 m, for example, it is known as *Engineer's scale*. According to this scale a specified distance on the map (*i.e.*, 1 cm) represents the corresponding distance on the ground (*i.e.*, 50 m). The other way is to indicate the scale by a ratio known as *representative fraction*, abbreviated as R.F. For the scale 1 cm = 50 m,

$$\text{R.F.} = \frac{1 \text{ cm}}{(50 \text{ m} \times 100) \text{ cm}} = \frac{1}{5000}$$

$$\text{or} \qquad = 1:5000$$

In *graphical representation*, a line subdivided into plan distance corresponding to convenient units of length on the ground, is drawn on the map as shown in Fig. 1.2.

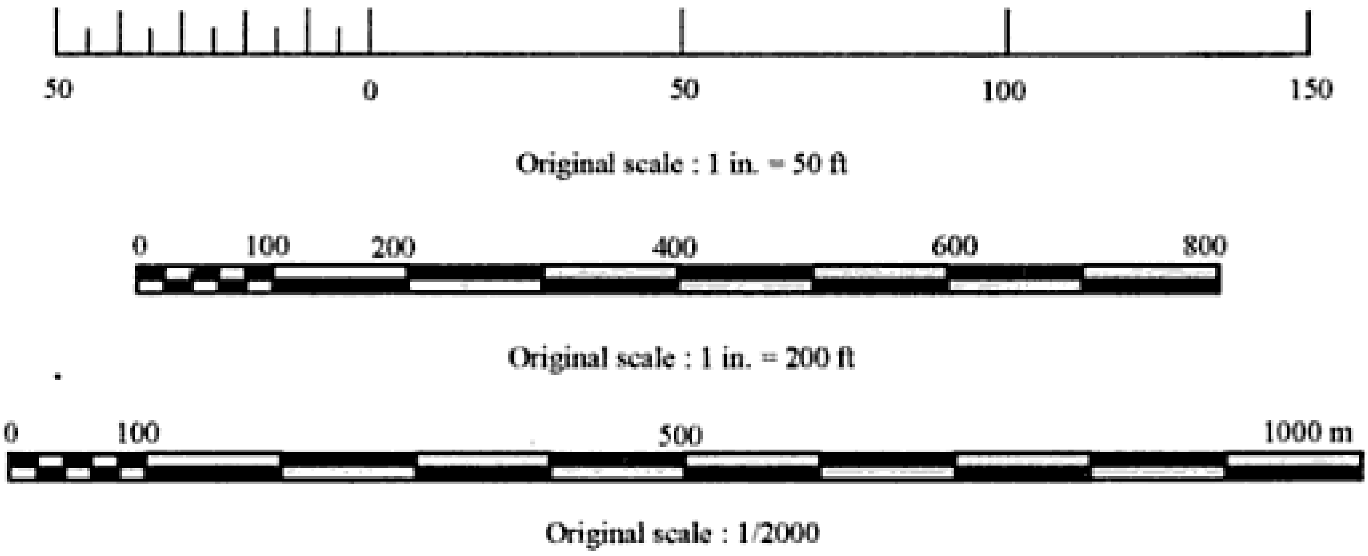


Fig. 1.2 Graphical scales

Since the distances on the maps can be determined by actual scaling, the graphical scale has advantage over the numerical scale that even when the map has been shrunk or has been reproduced to some other scale, they also change with the map, and therefore, the ratio of distances on the map to the distances on the ground are unaffected.

For detail plotting, the following two types of scales are used for linear measurements:

(*a*) Plain scale

(*b*) Diagonal scale.

Plain scale The *plain scale* is one which makes possible the measurement of some unit and its submultiples. Thus, if x is any unit then on a plain scale we can measure x and $(1/n)x$ where n is some suitable number. For example,

(*i*) Metre and one-tenth of a metre.

(*ii*) Kilometre and one-tenth of a kilometre.

(*iii*) 100 kilometre and one-fifth of 100 kilometres.

(*iv*) Hour and one-sixth of an hour.

If R.F. for a scale is 1:50 and maximum distance to be measured using the scale, is 6 m then to construct a plain scale, steps to be followed are

$$\begin{aligned} \text{50 units on the ground} &= \text{1 unit on the map} \\ \text{6 m on the ground} &= \frac{6}{50} \text{ m on the map} \\ &= \text{12 cm on the map} \end{aligned}$$

Draw a line of 12 cm, and divide it equally into 6 parts. The first part A on the left side is further divided into 10 equal parts to read $\frac{1}{10}$ m (Fig. 1.3).

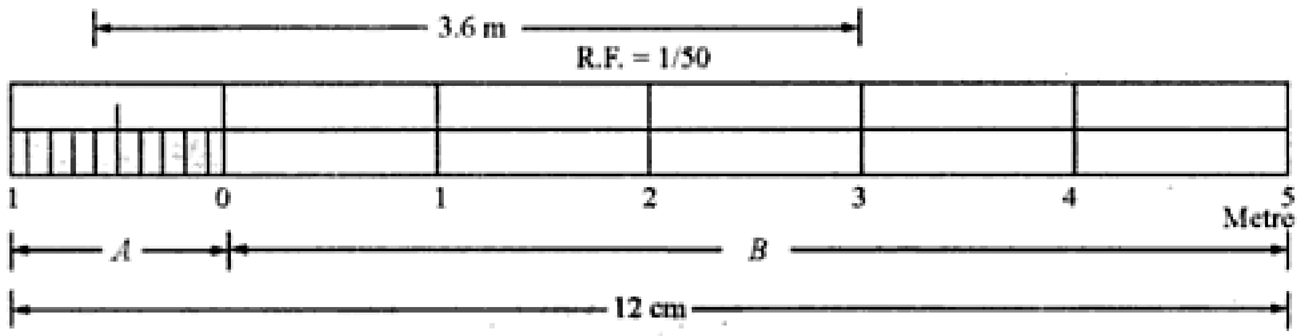


Fig. 1.3 Plain scale

Diagonal scale A *diagonal scale* reads some unit, its submultiples and further submultiples. Thus if x is any unit, the diagonal scale can measure x, $(1/n)x$ and $(1/m)(1/n)x$, where n and m are some suitable numbers. For example,

(*i*) Metre, one-tenth of a metre or decimetre, and one-tenth of decimetre.

(*ii*) Kilometres, 100 metres or hectometre and 10 metres or decametre.

The base of a diagonal scale is a plain scale where we have subdivisions in the part A, and the main unit of measurements in the part B as in a plain scale, shown in Fig. 1.3. To further divide the subdivisions of part A, help of diagonal is taken, as it is difficult to subdivide the divisions being small in length. This is explained in Fig. 1.4. In this figure, AB has been divided into 10 equal parts using the diagonal BC. The distances a_1b_1, a_2b_2, etc., are subdivisions of AB such that

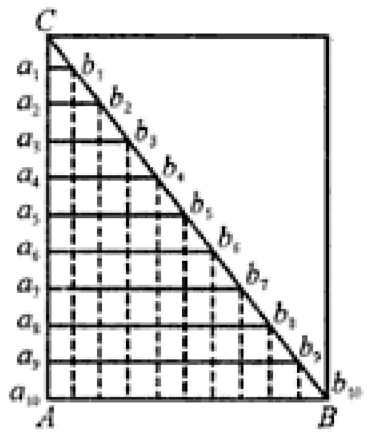


Fig. 1.4 Dividing a line with the help of a diagonal

$$a_1b_1 = \frac{1}{10}AB$$

$$a_2b_2 = \frac{2}{10}AB$$

$$a_3b_3 = \frac{3}{10}AB$$

$$\vdots \qquad \vdots$$

$$a_9b_9 = \frac{9}{10}AB$$

$$a_{10}b_{10} = \frac{10}{10}AB = AB.$$

Fig. 1.5 shows that the plain scale shown in Fig. 1.3 has been converted into a diagonal scale to read $\frac{1}{10}\left[\frac{1}{10}\text{ m}\right]$, *i.e.*, $\frac{1}{100}$ m or 1 cm by subdividing the divisions of part *A* of the plain scale with the help of diagonals for each division. A distance of 3.63 m has been marked in the figure.

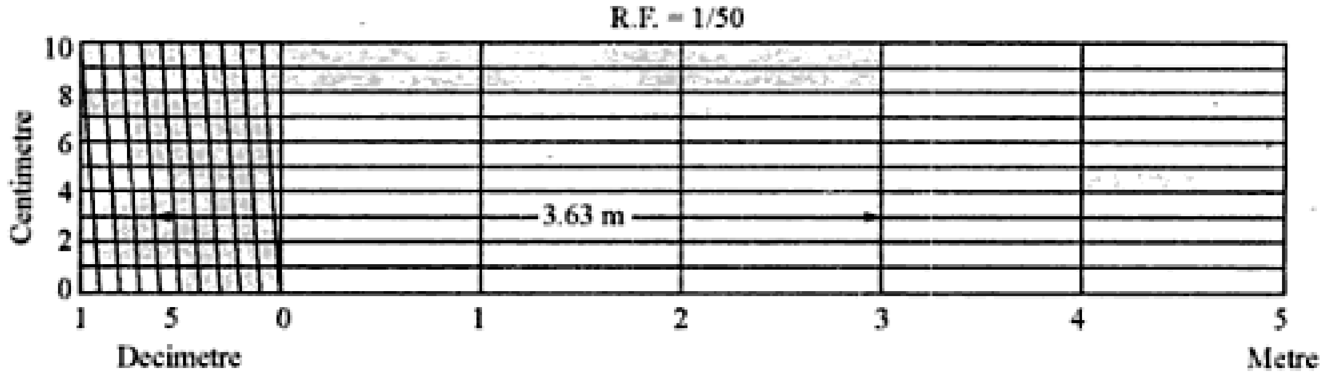


Fig. 1.5 Diagonal scale

ILLUSTRATIVE EXAMPLES

Example 1.1

Draw a plain scale of R.F. = 1/40 to show metres and decimetres, and long enough to measure up to 4 metres. Show a distance of 3.4 metres on it.

Solution (Fig. 1.6):

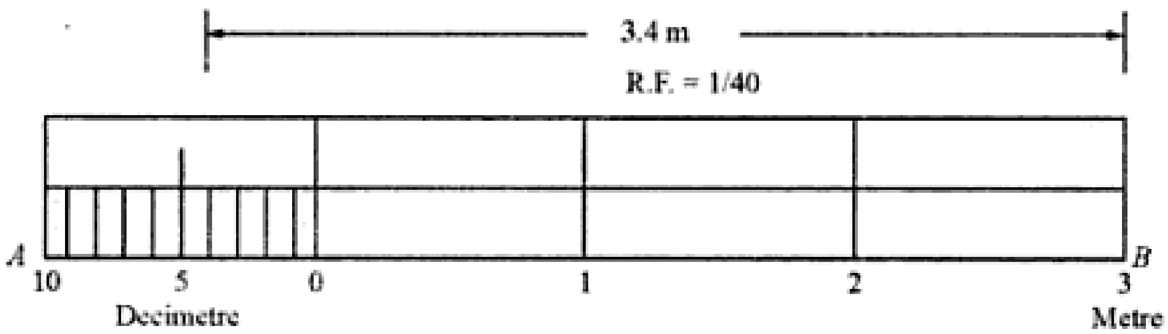


Fig. 1.6

$$\text{Length of scale} = \frac{1}{40}\times 4\text{ m} = 0.10\text{ m} = 10\text{ cm}$$

Draw a line *AB*, 10 cm long, and divide it into 4 equal parts, each part representing a single metre. Mark 0 (zero) at the end of the first division and then 1, 2 and 3, at the end of subsequent divisions as shown in the figure.

Subdivide the first division into 10 equal parts to represent single decimetre. Complete the scale, and mark the distance 3.4 m on it.

Example 1.2

A rectangular plot of 25 km^2 in area is shown on a map by a similar rectangular area of 1 cm^2. Construct a plain scale to show units of ten kilometres and single kilometres. The scale should be long enough to read up to 50 km. Measure a distance of 37 km on it.

Solution (Fig. 1.7):

$$1\text{ cm}^2 = 25\text{ km}^2$$

or $1 \text{ cm} = 5 \text{ km}$

$$\text{R.F.} = \frac{1 \text{ cm}}{5 \times 1000 \times 100 \text{ cm}} = \frac{1}{500000}$$

$$\text{Length of scale} = \frac{1}{500000} \times 50 \text{ km} = 10 \text{ cm}$$

Therefore, draw a line 10 cm long, and divide it into 5 equal parts, each representing 10 km. Subdivide the first part in 10 equal divisions, each representing 1 km. Complete the scale as shown in the figure, and mark the required distance 37 km on it.

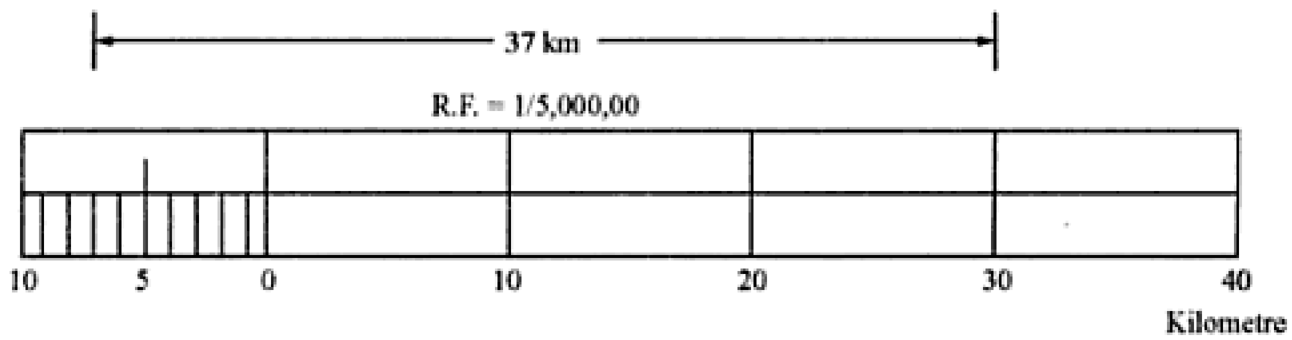


Fig. 1.7

Example 1.3

Draw a scale 1 cm = 3 m to read metres and decimetres, and to be long enough to read up to 30 m. Measure a distance of 22.3 m on it.

Solution (*Fig. 1.8*):

$$\text{R.F.} = \frac{1 \text{ cm}}{3 \times 100 \text{ cm}} = \frac{1}{300}$$

$$\text{Length of scale} = \frac{1}{300} \times 30 \text{ m} = 10 \text{ cm}$$

Draw a line of 10 cm length, and divide it into 3 equal parts, each division representing 10 m. Divide the first part into 10 equal parts, each division representing 1m. Then draw diagonals as shown in the figure to read 1/10 m and mark the distance 22.3 m on the scale.

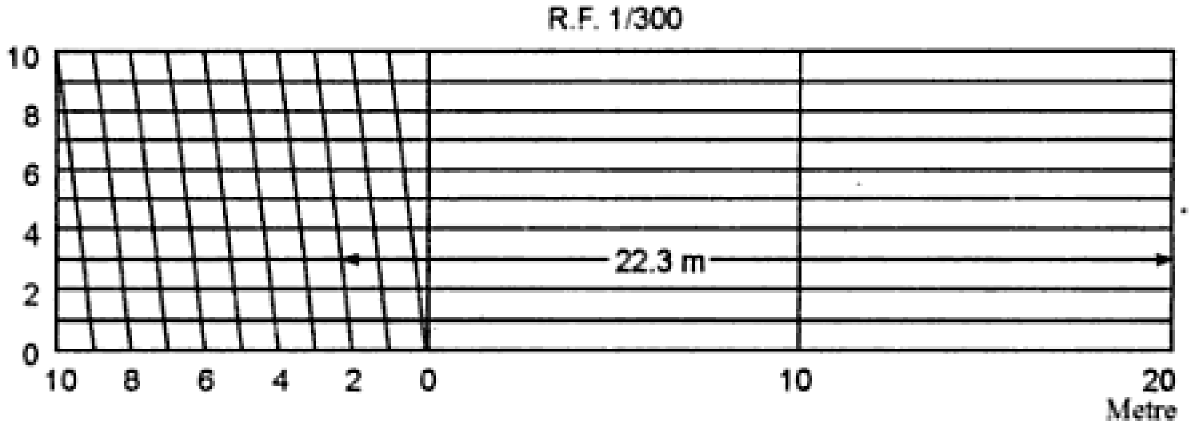


Fig. 1.8

1.8 PRECISION IN SURVEYING

The minimum distance which can be plotted on a map is 0.25 mm. The permissible error in linear measurements on the ground, thus, depends upon the scale of plotting. In other words, the error in linear
measurements which cannot be plotted on the map, is the permissible error. For example, if a map is required to be on scale 1 cm = 1 km, the error of $\frac{0.25 \times 1000}{10} = 25\,\text{m}$ will not affect the accuracy of the map. On the other hand, for the scale of 1 cm = 5 m of a map, the error more than $\frac{0.25 \times 50}{10} = 12.5$ cm cannot be tolerated.

The degree of precision in surveying mainly depends upon the purpose and scale of the map. Larger the scale better the precision required and *vice versa.*

1.9 DIFFERENCE BETWEEN A PLAN AND A MAP

The distinction between a plan and a map is rather arbitrary. It is difficult to say definitely that when a plan becomes a map.

Generally a plan is orthographic representation of features on or near the earth on a horizontal plane. Therefore, all the distances on a plan are horizontal distances to some scale. The earth's curvature is not taken into account, therefore, preferably the plans are prepared for smaller areas to avoid distortions due to curvature of the earth in the adopted projection system.

A map is also a graphical representation of features but differs from a plan when the scale is small and is constructed using a projection system other than orthographic. Generally, maps give some additional information such as about the topography with the help of contours.

PROBLEMS

1.1 What is surveying? Explain its importance in civil engineering.

1.2 What are broad divisions of surveying? State their objectives.

1.3 Discuss the classifications of surveying based on
(*a*) the function of survey, and
(*b*) instrument employed.

1.4 Explain the fundamental principles of surveying.

1.5 What are the various phases of survey work? Discuss each briefly.

1.6 Explain the difference between a plain scale and a diagonal scale.

2

SURVEY MEASUREMENTS, ERRORS AND PROPAGATION OF ERRORS

2.1 GENERAL

The purpose of surveying is essentially to locate the positions of points on or near the surface of earth by making field measurements of distances and angles. Often, these basic field measurements are used to calculate other quantities of direct interest such as areas and volumes. The measurements always contain errors due to human limitations, imperfection in instruments, environmental changes, or carelessness on the part of the observer. Since the calculated quantities are evaluated through relationships with the measured quantities, the errors in measured quantities get propagated into the calculated quantities.

The surveyors should understand the different types of errors, their sources and how they propagate. He should know how to minimise the errors while taking the measurements. The errors which still creep in should be eliminated or their effects should be minimised as far as possible. He should also be able to find the errors in the calculated quantities.

2.2 RELIABILITY OF MEASUREMENTS

Several terms are used to express the reliability of measurements. The two most common are

1. Precision
2. Accuracy.

Precision is the degree of closeness or conformity of repeated measurements of the same quantity to each other whereas the *accuracy* is the degree of closeness or conformity of a measurement to its true value.

It is perfectly feasible to have a data set that is highly accurate but not very precise, and vice versa. The difference between accuracy and precision is best illustrated in Fig. 2.1, that is results of a shooting competition between four contestants *A*, *B*, *C*, and *D*. The shooting results indicate that

(*i*) Contestant *A* is accurate but not precise,
(*ii*) Contestant *B* is inaccurate but not precise,
(*iii*) Contestant *C* is neither accurate nor precise, and
(*iv*) Contestant *D*, the winner, is both accurate and precise,

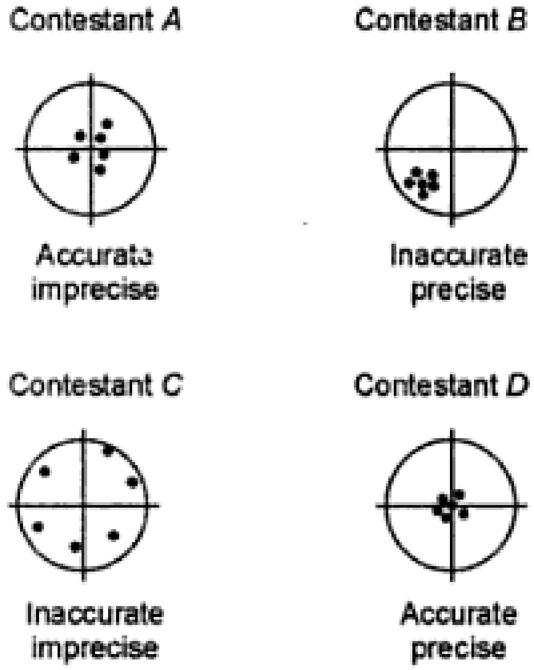


Fig. 2.1 Accuracy versus precision

Significant Figures

The number of digits or *significant figures* is an indication of the accuracy of a measurement. Only these digits in a measured number have significance. The number of significant figures in a numerical quantity equals "the number of digits in the quantity less all zero digits that are used to fix the position of the decimal point." The following examples make this clear.

Two significant figures : 32, 3.2, 0.32, 0.0032, 0.030
Three significant figures : 131, 13.1, 0.000131, 0.0320
Four significant figures : 3456, 34.56, 0.00003456, 32.00

The significant figures are very important from the following point of views.

(*i*) If large number of operations are required in computations, unnecessary digits retained after significant figures will take extra computation time resulting into wastage of time and money.

(*ii*) On the other hand, if one or two digits extra than required ones are not used in computations, the final computed values of a quantity are badly affected due to rounding off errors.

Rounding Off Numbers

Dropping one or more digits so that only significant ones or those that are necessary and sufficient for subsequent calculations, is known as *rounding off number*. If more digits have been dropped than the required ones, the result will have rounding off errors.

For example $\frac{1}{7}$ is equal to 0.143.........is in error by 0.0001428...............

The rules given below should be followed in rounding off numbers:

Rule 1: If the digit to be dropped is less than 5, it is ignored. For example, 38.274 is rounded off to 38.27.

Rule 2: If the digit to be dropped is greater than 5, increase the preceding digit by one. For example, 38.276 is rounded off to 38.28.

Rule 3: If the digit to be dropped is 5, the most common practice, which produces balanced result, is to use nearest even number for the previous digit, *e.g.*, 38.275 will become 38.28 and 38.285 will also be taken as 38.28.

2.3 CONCEPT OF ERRORS IN MEASUREMENTS

Different conditions discussed in Sec.2.1 under which the measurements are made, cause variations in measurements, and therefore, no measured quantity is completely determinable. A fixed value of a quantity concieved as its *true value*, can never be measured, as some errors will always creep in during its measurement, and what we get is nothing more than an *estimate* of the true value.

The difference between the measured quantity x and its true value τ is known as *error* ε, *i.e.*,

$$\varepsilon = x - \tau \qquad \text{...(2.1)}$$

Mathematically the measurement or observation is a variable, and study of observational errors and their behaviour is essentially the study of observations itself. *Theory of errors* deals with the study of observational errors.

Since the true value of a measurement cannot be determined, the exact value of ε can never be found out. However, if we can obtain a good estimate $\hat{x}$ of τ, then $\hat{x}$ can be used as a reference to express the variation in observed values x. Therefore, if we define υ as *residual*, then

$$\upsilon = \hat{x} - x \qquad \text{...(2.2)}$$

The residuals express the variations in the measurements. The quantity ($\hat{e}$) equal to ($-\upsilon$) is called *correction* to the measured quantity or *residual error, i.e.*,

$$(-\hat{e}) = \hat{x} - x$$

or Residual $= \hat{x} - x$

and residual error $= x - \hat{x}$.

2.4 TYPES OF ERRORS

Errors have been traditionally classified into the following three types:

1. Gross errors (blunders or mistakes),
2. Systematic errors, and
3. Random or accidental errors.

Gross Errors

Gross errors are, in fact, not errors at all, but results of mistakes that are due to the carelessness of the observer. Pointing on the wrong survey target, incorrect reading on the scale, reading on the wrong scale, and wrong recording the value of reading, are some examples of blunders or mistakes. The blunders or mistakes generally result into large errors which can easily be detected compared to small errors.

Good field procedures are designed to assist in detecting the gross errors. These may be

(*a*) Careful checking of all pointings on survey targets.

(*b*) Taking multiple readings on scales and checking for reasonable consistency.

(*c*) Verifying recorded data by reading scales.

(*d*) Taking repeated measurements independently and checking for consistency.

(*e*) Using simple geometric and algebraic checks such as sum of all the angles taken at a station is 360°, sum of three angles in a plane triangle is 180°.

The gross errors must be detected and eliminated from the survey measurements before such measurements can be used.

Systematic Errors

The errors which are systematic or follow some pattern, are termed as *systematic errors*. Such errors are always according to some deterministic system and can be expressed by functional relationship. Measurement of distance by a too short tape and expansion of steel tape with respect to temperature, are the examples of systematic errors. Such errors are of a constant nature and hence can be computed using the functional relationships, and the measurement can be corrected for them.

The personal bias of an observer, and imperfect conditions of the instruments also lead to systematic errors. Systematic errors also occur through simplification of geometry such as considering a plane triangle instead of spherical triangle.

Like the gross errors, the systematic errors must also be detected and eliminated from the survey measurements before adjusting the remaining errors.

Random Errors

After all mistakes are detected and removed, and the measurements are corrected for all known systematic errors, there will still remain some variations in measurements that result from observational errors. These errors do not have any known functional relationship based on a deterministic system and have random behaviour, and therefore, are known as *random errors*. As stated earlier that a measurement is looked upon mathematically as a variable, the measurement having a component of error which behaves like a random variable, itself becomes a random variable.

Random errors, often called as accidental errors, are unpredictable in regard to both size and algebraic sign. They are truely accidental and cannot be avoided. The general behaviour of the random errors is that

(*i*) a plus error will occur as frequently as will a minus error,
(*ii*) small errors will occur more frequently than large errors, and
(*iii*) very large errors do not occur at all, or their chance to occur is remote.

Whereas systematic errors are dealt with mathematically using functional relationships or models, random errors use probability models.

Theory of errors discussed in Chapter 2 of *Higher Surveying*, deals with random errors.

2.5 CONCEPT OF PROBABILITY IN SURVEY MEASUREMENTS

A survey measurement finally left with only random errors, is a random variable just as number of dots on the top face of a dice. In order to understand the random nature of a survey measurement, it may be measured repeatedly and its frequency distribution describing its random variation is studied. A distance of about 510 m is measured 200 times, and the measurements free of gross errors and corrected for systematic errors, are presented in Table 2.1.

Table 2.1 Repeated measurements of a distance

Measured values of distance (m)	Number of measurements (frequency = f)	Relative frequency ($R = f/\Sigma f$)
510.11	1	0.005
510.12	3	0.015
510.13	7	0.035
510.14	18	0.090
510.15	21	0.105
510.16	35	0.175
510.17	39	0.195
510.18	30	0.150
510.19	23	0.115
510.20	10	0.050
510.21	11	0.055
510.22	0	0.000
510.23	2	0.010
	$\Sigma f = 200$	$\Sigma R = 1$

In Table 2.1, the relative frequency for a given interval is an indication of probability implied by the following equation.

$P(E)$ [probability of a favourable outcome]

$$= \frac{f \text{ [number of favourable outcomes]}}{n \text{ [number of all possible outcomes]}} \quad \text{...(2.3)}$$

For example, for the inclusive interval 510.16 m to 510.169 m there are $f = 35$ observations. Therefore, the corresponding relative frequency is

$$\frac{f}{n} = \frac{35}{200} = 0.175$$

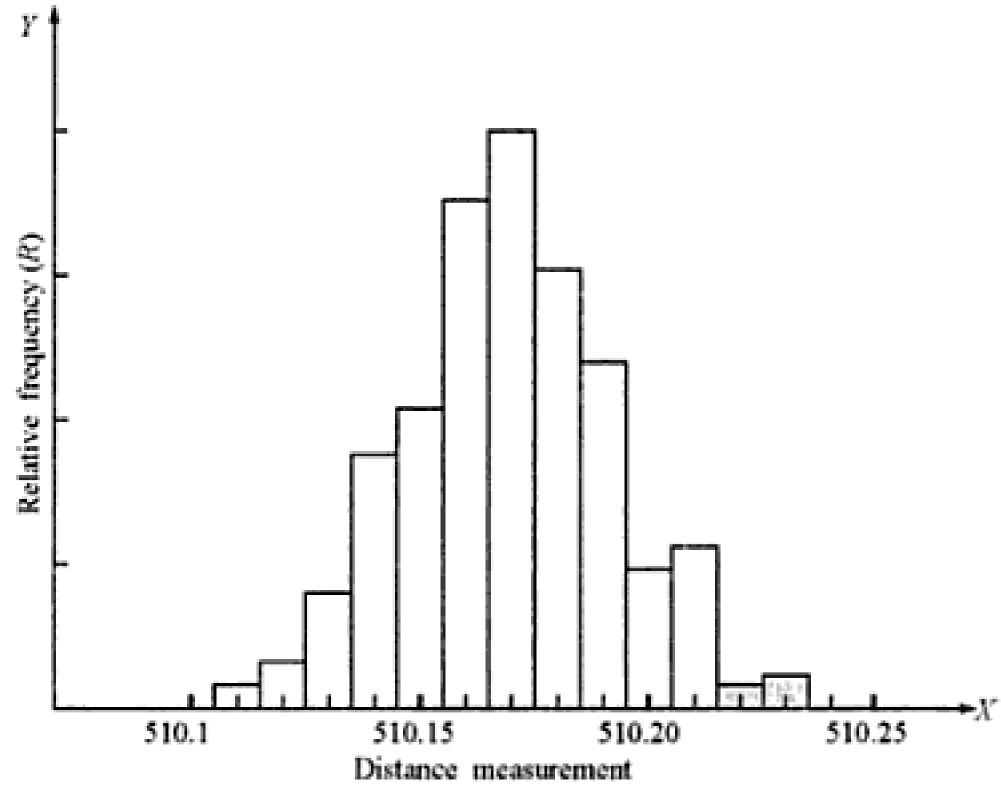


Fig. 2.2 Histogram

A diagram (Fig. 2.2), known as *frequency distribution* or more frequently *histogram*, can be constructed between the given class intervals and corresponding relative frequencies. It is found that the frequency distribution is centered on or near the value 510.175 m which is mean of all the readings. The highest frequencies of occurrence are at or near the central value. An interesting point to note is that the mean value has the highest relative frequency.

If the number of repeated measurements are increased to a very large number, and the class interval is made narrow, it will be found that each relative frequency approaches to a stable limit which is the *probability*. In such case the histogram approaches to the appearance of a curve, known as *probability distribution curve* or *probability density function curve* shown in Fig. 2.3. The area under the curve between two measured values of a distance, x_1 and x_2, is the probability of a measurement falling be

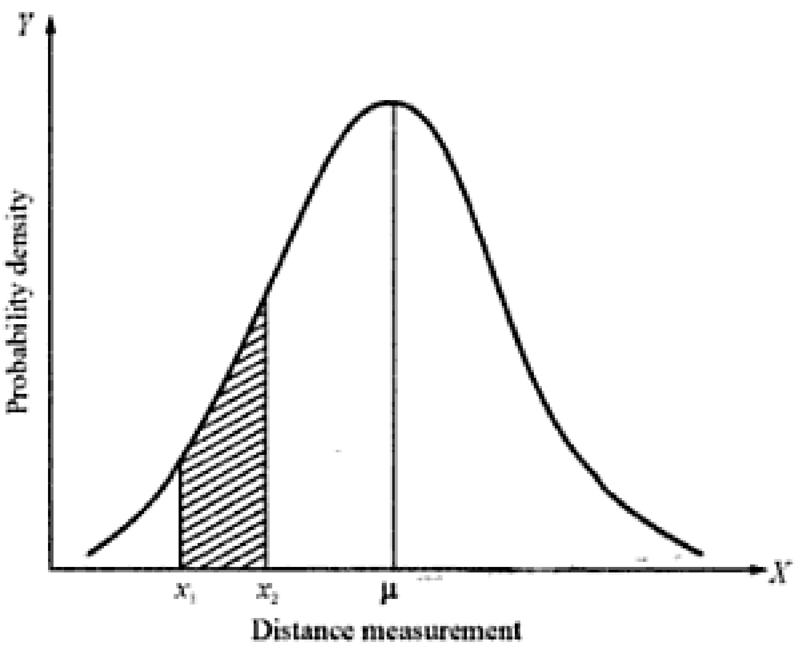


Fig. 2.3 Probability density function curve

tween the values x_1 and x_2. The central value μ represents the *distribution mean value* of the distance measurements. The distribution mean μ is taken as representative of the true value of the measured distance.

2.6 NORMAL DISTRIBUTION

The statistical analysis of survey measurements has indicated that the survey measurements follow *normal distribution* or *Gaussian distribution*. The normal distribution is expressed by

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\upsilon}{\sigma}\right)^2} \qquad \text{...(2.4)}$$

where y = the frequency of occurrence of residual,

υ = the residual,

σ = the standard deviation, and

e = the base of the natural logarithm (e = 2.718).

Fig. 2.4 shows a normal distribution curve.

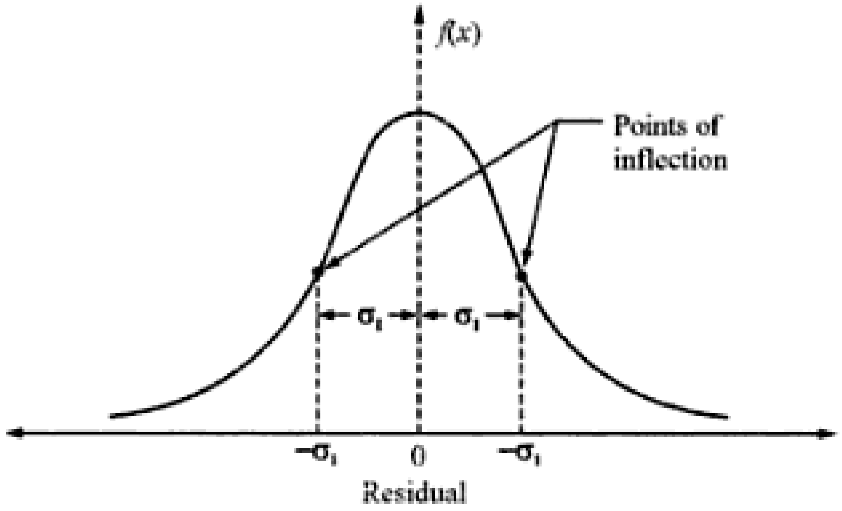


Fig. 2.4 Normal distribution curve

If the abscissa interval which is designated as the *class interval*, be denoted by I, and n be the number of measurements or residuals, then Eq. (2.4) takes the form

$$y = \frac{nI}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\upsilon}{\sigma}\right)^2} \qquad \text{...(2.5)}$$

The *standard deviation* σ, also known as *standard error*, is defined as

$$\sigma = \pm\sqrt{\frac{\Sigma\upsilon^2}{n-1}} \qquad \text{...(2.6)}$$

where n = number of observations.

The normal density function is characterized by two parameters, the distribution mean μ, and the standard deviation σ. For a large number of observations μ of residuals υ becomes zero. σ is also termed as *root-mean square* (R.M.S.) *error*. The square of standard deviation σ^2 is called the *variance*.

The standard deviation is a measure of the spread or dispersion of the probability distribution, and reflects the degree of variation in the measurement. If the repeated measurements are closely clustered together as shown in Fig. 2.5, they are said to have high precision and if they are widely spread apart, they have low precision.

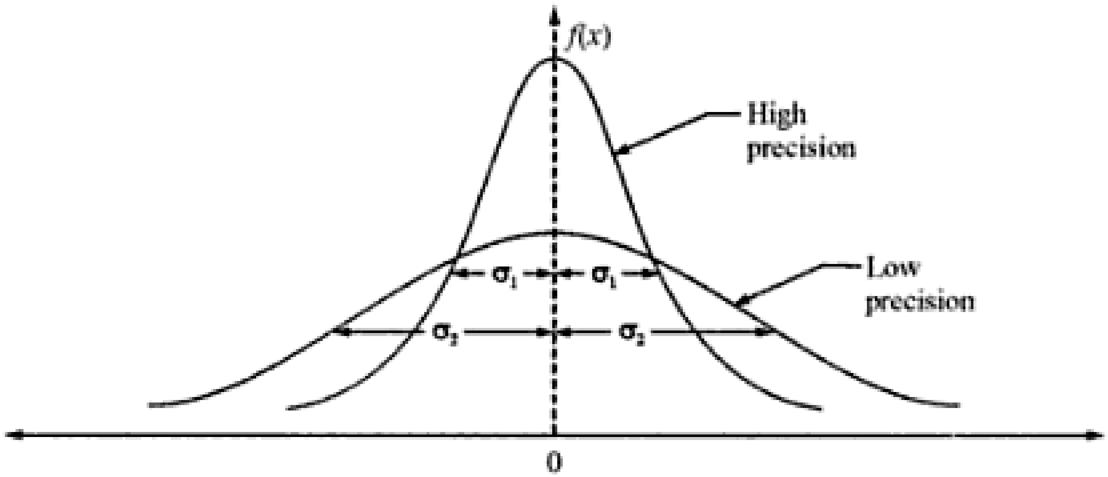


Fig. 2.5 Precision of measurements

There are some other indicators of precision, and are briefly given below.

Variance which is expressed as

$$V = \sigma^2$$

$$= \frac{\Sigma \upsilon^2}{n-1} \qquad ...(2.7)$$

is also used as a measure of dispersion or spread.

Standard error of the mean expressed by

$$\sigma_m = \pm\sqrt{\frac{\Sigma \upsilon^2}{n\,(n-1)}}$$

$$= \pm\frac{\sigma}{\sqrt{n}} \qquad ...(2.8)$$

is indicator of the limits of the errors found within which the true value of the mean will lie with a certainty of 68.3% of its being correct.

Standard error of the single measurement expressed by

$$\sigma_1 = \pm\sqrt{\frac{\Sigma \upsilon^2}{n-1}} \qquad ...(2.9)$$

is equal to standard *error* σ. The two terms are used synonymously.

Most probable error expressed by

$$e = \pm\, 0.6745\sqrt{\frac{\Sigma \upsilon^2}{(n-1)}}$$

$$e = \pm\, 0.6745\, \sigma \qquad ...(2.10)$$

is defined as the error for which there are equal chances of its being less than and more than the probable error. It is also called 50% error and expressed as E_{50}.

Most probable error of the mean is expressed as

$$e_m = \pm 0.6745 \sqrt{\frac{\Sigma \upsilon^2}{n(n-1)}}$$

$$= \pm\ 0.6745\sigma_m \qquad ...(2.11)$$

Maximum error corresponds to $\pm$ 3.29 σ. It is often used to separate mistakes or blunders from the random errors. If any error deviates from the mean by more than the maximum error, it is considered as a gross error, and that measurement is rejected. Different percentage errors are also sometimes used. They are expressed as

(*i*) E_{90} (90% error) $= \pm\ 1.645\sigma$ ...(2.12)
(*ii*) E_{95} (95% error) $= \pm\ 1.96\sigma$ ...(2.13)
(*iii*) $E_{95.5}$ (95.5% error) $= \pm\ 2.0\sigma$ ...(2.14)
(*iv*) $E_{99.7}$ (99.7% error) $= \pm\ 3.0\sigma$ ...(2.15)

ILLUSTRATIVE EXAMPLES

Example 2.1

The distance between two points *A* and *B* was measured 10 times under identical conditions, and the measured values were recorded as

53.56 m, 53.52 m, 53.54 m , 53.58 m, 53.55 m, 53.60 m, 53.54 m, 53.57 m, 53.54 m, 53.56 m.

Determine the following:

(*i*) The standard error
(*ii*) The standard error of the mean
(*iii*) The most probable error
(*iv*) The most probable error of the mean
(*v*) The variance
(*vi*) The maximum error.

Solution:

Determine the mean μ of the measurements, residuals υ, and υ^2 as shown in Table 2.2.

Number of observations $n = 10$

(*i*) Standard deviation

$$\sigma = \pm \sqrt{\frac{\Sigma \upsilon^2}{n-1}}$$

$$= \pm \sqrt{\frac{4.800 \times 10^{-3}}{10-1}} = \pm\ \mathbf{0.023\ m.}$$

Table 2.2

Measurements (x) (m)	Residuals ($\upsilon = \mu - x$)	υ^2
53.56	– 0.004	1.600×10^{-5}
53.52	0.036	1.296×10^{-3}
53.54	0.016	2.560×10^{-4}
53.58	– 0.024	5.760×10^{-4}
53.55	0.006	3.600×10^{-5}
53.60	– 0.044	1.936×10^{-3}
53.54	0.016	2.560×10^{-4}
53.57	– 0.014	1.960×10^{-4}
53.54	0.016	3.600×10^{-5}
53.56	– 0.004	1.960×10^{-4}
$\mu = \dfrac{535.56}{10} = 53.556$	$\Sigma\upsilon = -0.000$	$\Sigma\upsilon^2 = 4.800 \times 10^{-3}$

(*ii*) Standard error of the mean

$$\sigma_m = \pm\sqrt{\frac{\Sigma\upsilon^2}{n(n-1)}}$$

$$= \pm\sqrt{\frac{4.800\times10^{-3}}{10\times(10-1)}} = \pm\ \mathbf{0.007\ m.}$$

(*iii*) Most probable error

$$e = \pm 0.6745\sigma$$

$$= \pm 0.6745 \times 0.023 = \mathbf{\pm 0.016\ m.}$$

(*iv*) Most probable error of the mean

$$e_m = \pm\ 0.6745\ \sigma_m$$

$$= \pm\ 0.6745 \times 0.007 = \mathbf{\pm 0.005\ m.}$$

(*v*) Variance

$$V = \sigma^2$$

$$= (\pm\ 0.023)^2 = \mathbf{0.0005\ m^2.}$$

(*vi*) Maximum error

$$e_{max} = \pm\ 3.29\sigma$$

$$= \pm\ 3.29 \times 0.023 = \pm\ \mathbf{0.076\ m.}$$

Example 2.2

The following eight measurements of a horizontal angle were made:

S.No.	Horizontal angle (A)
1	32°22′10″
2	32°21′50″
3	32°21′40″
4	32°22′00″
5	32°21′50″
6	32°22′00″
7	32°22′10″
8	32°22′20″

Compute the standard deviation, the most probable error, and the maximum error.

Solution:

The mean, $\mu = \dfrac{\Sigma A}{n} = 32°22'00''$

The residuals can be computed from

$$\upsilon = \mu - A$$

$$\upsilon_1 = +10'', \upsilon_2 = -10'', \upsilon_3 = -20'', \upsilon_4 = 0'', \upsilon_5 = -10'', \upsilon_6 = 0'', \upsilon_7 = +10'', \upsilon_8 = +20''$$

$\Sigma\upsilon^2 = 1200$

The standard deviation

$$\sigma = \sigma = \pm\sqrt{\frac{\Sigma\upsilon^2}{n-1}} = \pm\sqrt{\frac{1200}{7}}$$

$$= \pm\ \mathbf{13.1''}.$$

The most probable error of the mean

$$e = \pm\ 0.6745\sigma_m$$

$$= \pm 0.6745\frac{\sigma}{\sqrt{n}}$$

$$= \pm 0.6745 \times \frac{13.1}{\sqrt{8}}$$

$$= \pm\ \mathbf{3.1''}$$

The maximum error

$$
\begin{aligned}
e_{max} &= \pm\ 0.29\sigma \\
&= \pm\ 3.29 \times 13.1 \\
&= \pm\ \mathbf{43.1''}.
\end{aligned}
$$

2.7 PROPAGATION OF ERROR

In surveying the two basic quantities which are measured, are angles and distances. Often these basic field measurements are used to calculate other quantities such as horizontal distances from measured slope distances, elevations from measured difference in height, areas, volumes, etc. The calculation of other quantities using the measured quantities is done through mathematical relationship between them. Since the measured quantities have errors, it is inevitable that the quantities computed from them will not have errors. Evaluation of the errors in the computed quantities as functions of the errors in the measurements, is called *error propagation*.

Let y be a quantity to be computed using a measured quantity x which has error dx through the relationship

$$y = ax + b \qquad \text{...(2.16)}$$

The coefficient a and the constant b are assumed to be free of errors.

Now if x_t and y_t are the true values of x and y, respectively, then

$$x = x_t + dx \qquad \text{...(2.17)}$$

$$y_t = ax_t + b \qquad \text{...(2.18)}$$

But from Eq. (2.16) and Eq. (2.17)

$$
\begin{aligned}
y &= a\ (x_t + dx)\ +\ b \\
&= ax_t + adx\ +\ b \\
&= (ax_t + b)\ +\ adx \\
&= y_t + adx
\end{aligned}
$$

If dy is the error in y, then from Fig. 2.6, we get

$$\tan\theta = \frac{dy}{dx} = a$$

or $$dy = adx \qquad \text{...(2.19)}$$

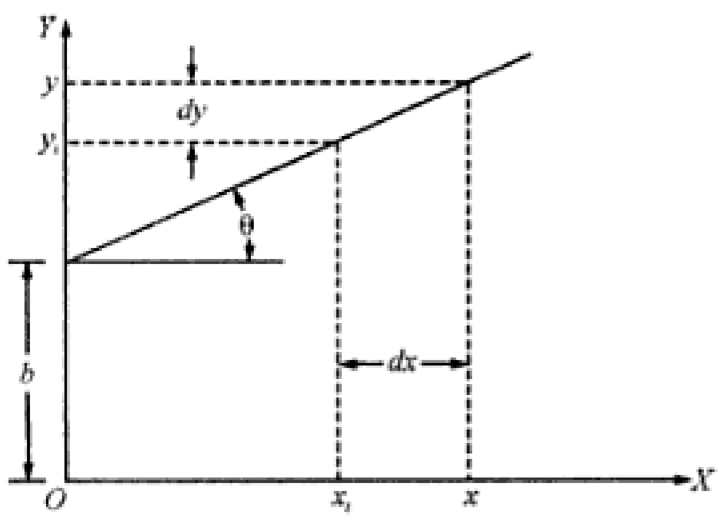


Fig. 2.6 Propagation of error in linear function

or $$dy = \left(\frac{dy}{dx}\right)dx \qquad ...(2.20)$$

The Eq. (2.19) or (2.20) gives the error dy in the computed quantity y. In Eq. (2.19), the error function $dy = adx$ is linear in error dx in identical manner to $y = ax + b$. This will not be true in case of nonlinear functions such as $y = x^2$.

Let us take an example of a nonlinear function.

$$y = x^2 \qquad ...(2.21)$$

$$y_t = x_t^2$$

or $$y = (y_t + dy) = (x_t + dx)^2$$

or $$y_t + dy = x_t^2 + 2x_t dx + (dx)^2$$

or $$dy = 2x_t dx + (dx)^2 \qquad ...(2.22)$$

Neglecting $(dx)^2$ as dx is a small quantity, we get

$$dy = 2x_t dx \qquad ...(2.23)$$

$$\frac{dy}{dx} = 2x_t$$

From Eq. (2.23), we find that $2x_t$ is the derivative of y with respect to x, evaluated at $x = x_t$.

$$\left(\frac{dy}{dx}\right)_{x=x_t} = 2x_t$$

Thus, from Eq. (2.22)

$$dy = \frac{dy}{dx}dx + (dx)^2 \qquad ...(2.24)$$

Again neglecting $(dx)^2$, we get

$$dy = \frac{dy}{dx}dx \qquad ...(2.25)$$

with the assumption that $(dx)^2$ is negligible.

Propagation of Standard Error

If the standard errors in the measured quantities are known, the standard error in the computed quantity can be evaluated. Let the standard errors in the measured quantities x_1, x_2, and x_3 be σ_{x_1}, σ_{x_2}, and σ_{x_3}, respectively. The quantity y is computed from x_1, x_2, and x_3.

Thus $y = f\ (x_1,\ x_2,\ x_3)$

The error in y is given by

$$dy = \frac{\partial f}{\partial x_1}dx_1 + \frac{\partial f}{\partial x_2}dx_2 + \frac{\partial f}{\partial x_3}dx_3 \qquad ...(2.26)$$

and the standard error in y is obtained from the following equation.

$$\sigma_y^2 = \left[\frac{\partial f}{\partial x_1}\sigma_{x_1}\right]^2 + \left[\frac{\partial f}{\partial x_2}\sigma_{x_2}\right]^2 + \left[\frac{\partial f}{\partial x_3}\sigma_{x_3}\right]^2 \qquad ...(2.27)$$

ILLUSTRATIVE EXAMPLES

Example 2.3

The dimensions of a trapezoidal land parcel is shown in Fig. 2.7. What is the ordinate y for a measured distance $x = 37.325$ m? If the error in the measured distance is 0.015 m, what would be the corresponding error in the calculated distance of y ?

Solution:

Let us represent

$$y = ax + b$$

From Fig. 2.7, we have

$$a = \tan\theta = \frac{EC}{DE} = \frac{80-50}{100} = 0.3 \text{ (exact)}$$

and $\quad b = AD = 50$

As a and b have exact values, therefore, they are errorless

$$y_{(x=37.325)} = 0.3 \times 37.325 + 50$$
$$= \mathbf{61.1975\ m}.$$

and error in y for $dx = 0.015$ m, therefore,

$$dy = a.dx$$
$$= 0.3 \times 0.015$$
$$= \mathbf{0.0045\ m}.$$

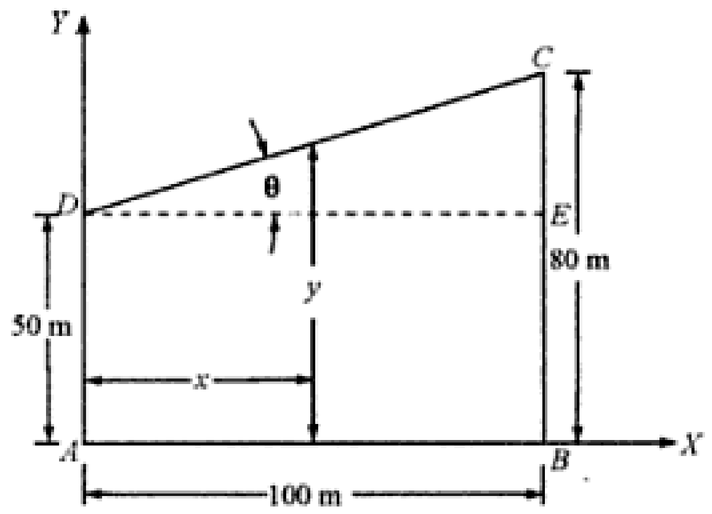


Fig. 2.7

Example 2.4

A measured side of a square piece of land is 140.140 m, and is in error by 0.030 m. What is the corresponding error in the computed area of the land?

Solution:

If x is the length of the side, the area is given by

$$y = x^2$$

$$\frac{dy}{dx} = 2x$$

$$\left(\frac{dy}{dx}\right)_{x=140.140} = 2 \times 140.140 = 280.28$$

Error in y will be

$$\begin{aligned} dy &= \frac{dy}{dx} dx \\ &= 280.28 \times 0.030 \text{ m} \\ &= \mathbf{8.408\ m^2.} \end{aligned}$$

Example 2.5

The measured sides of a rectangular tract are 30.270 m and 56.070 m. The measurement was made using a 30 m metallic tape, too short by 0.025 m. What would be the error in the area of the tract.

Solution:

Let the two sides be x_1 = 30.270 m and x_2 = 56.070 m then the area

$$y = x_1 x_2 \qquad \ldots (a)$$

or $$dy = \frac{\partial y}{\partial x_1} dx_1 + \frac{\partial y}{\partial x_2} dx_2 \qquad \ldots (b)$$

Now from Eq. (*a*), we have

$$\frac{\partial y}{\partial x_1} = x_2 = 56.070 \text{ m}$$

$$\frac{\partial y}{\partial x_2} = x_1 = 30.270 \text{ m}$$

Errors dx_1 in x_1, and dx_2 in x_2 are computed as

$$dx_1 = \frac{0.025}{30} \times 30.270 = 0.0252 \text{ m}$$

and $dx_2 = \frac{0.025}{30} \times 56.070 = 0.0467 \text{ m}$.

Therefore, from Eq. (*b*), the error in the area is

$$dy = 56.070 \times 0.0252 + 30.270 \times 0.0467$$
$$= \mathbf{2.827\ m^2.}$$

Example 2.6

The angle α is related to the angles β and γ as below:

$$\alpha = 3\beta\gamma$$

The observed values of β and γ are 30″ and 18″, respectively. If $\sigma_\beta = \pm\ 0.02''$ and $\sigma_\gamma = \pm\ 0.04''$, determine σ_α.

Solution:

It is given that

$$\alpha = 3\beta\gamma$$

From Eq. (2.27), we have

$$\sigma_\alpha^2 = \left[\frac{\partial\alpha}{\partial\beta}\sigma_\beta\right]^2 + \left[\frac{\partial\alpha}{\partial\gamma}\sigma_\gamma\right]^2$$

$$= \left[\frac{\partial\,(3\beta\gamma)}{\partial\beta}\sigma_\beta\right]^2 + \left[\frac{\partial(3\beta\gamma)}{\partial\gamma}\sigma_\gamma\right]^2$$

$$= (3\gamma\sigma_\beta)^2 + (3\beta\sigma_\gamma)^2$$

$$\sigma_\alpha = \sqrt{(3\times 18\times 0.02)^2 + (3\times 30\times 0.04)^2}$$

$$= \pm\ \mathbf{3.76''.}$$

PROBLEMS

2.1 Write a brief note on the role of errors in survey measurements.

2.2 What do you understand by significant figures and rounding off numbers? How these affect the computations?

2.3 Write a short note on the classification of errors. Discuss each giving suitable examples.

2.4 Discuss the application of theory of probability to the random errors.

2.5 Write a note on the normal distribution.

2.6 Differentiate between precision and accuracy.

2.7 Explain the following terms

(*a*) Survey Measurement, (*b*) True value, (*c*) True error, (*d*) Residual, (*e*) Standard deviation, (*f*) Variance, (*g*) Standard error of the mean, (*h*) Standard error of the single measurement, (*i*) Most probable error.

2.8 Write a brief note on propagation of errors.

2.9 The difference in elevation Δh between two fixed points was measured repeatedly for a total of 16 times with the following results:

n	Δh (m)	n	Δh (m)
1	7.8621	9	7.8631
2	7.8632	10	7.8641
3	7.8630	11	7.8630
4	7.8646	12	7.8640
5	7.8642	13	7.8630
6	7.8652	14	7.8637
7	7.8620	15	7.8633
8	7.8638	16	7.8630

Compute the mean and standard error of the mean of these observations.

2.10 The following independent readings in seconds, were taken in order to determine the precision of the setting of the micrometer of 1″ direction theodolite:

52.9	54.9	54.1	52.1	54.1
55.0	52.9	55.4	51.5	53.4
53.9	52.1	53.5	52.1	51.0
54.9	54.1	50.8	50.3	54.6
54.1	50.2	50.0	55.9	53.1
54.2	51.9	52.4	51.0	53.9
52.3	51.3	53.2	51.6	53.1
53.0	52.1	50.3	51.9	53.6
52.2	54.6	51.1	51.1	54.9
51.6	51.4	49.9	53.6	54.1
53.5	53.7	50.3	53.0	52.6
55.4	54.4	51.6	51.7	48.9
54.9	51.8	49.2	56.3	52.7
50.7	48.2	50.6	52.2	53.1
52.5	50.7	53.1	57.1	51.9
52.3	54.2	52.9	51.6	50.9
52.9	54.0	54.3	50.1	51.9
51.3	53.8	53.3	54.2	54.1
55.9	55.1	47.4	56.2	53.2
48.8	52.9	50.5	53.9	53.5

(*i*) Plot a histogram for these readings, with a class interval of 1″, beginning with 47″.

(*ii*) Compute the mean value of the readings, the standard error, and standard error of the mean.

(*iii*) Using the standard error of the set of readings, compute ordinates of the normal distribution curve representing the set.

(*iv*) Plot the curve on the histogram in (*i*).

(*v*) Locate the positions of $+\sigma$ and $-\sigma$ on the curve in (*iii*).

2.11 The volume of a pyramid is 1/3 b^2h. The base of the pyramid is a square of measured side b as 30.4 ± 0.8 cm. The height h of the pyramid is measured as 25.0 ± 0.05 cm. Compute the volume of the pyramid and the standard error of the computed volume.

2.12 Calculate the standard error of the volume of a cuboid whose sides x, y, and z have the values and standard errors as under:

$x = 60 \pm 0.03$ cm

$y = 50 \pm 0.02$ cm

$z = 40 \pm 0.01$ cm.

2.13 Calculate the area of the field shown in Fig. 2.8, and its standard error, if the measured values of a and b with their standard errors are as under:

$a = 102.10 \pm 0.02$ m

$b = 260.75 \pm 0.03$ m.

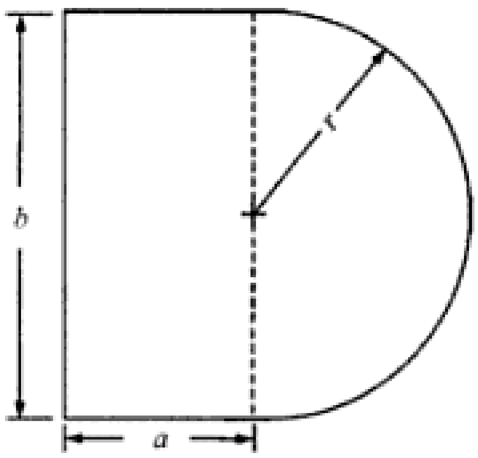


Fig. 2.8

2.14 Three sides of a triangle were measured as $a = 248.25 \pm 0.04$ m, $b = 107.10 \pm 0.02$ m, and $c = 336.65 \pm 0.05$ m. Compute the angles in the triangle with their standard errors.

2.15 Compute the area of the triangle in Problem 2.14 and its standard error.

2.16 The two sides and the included angle of a triangle are measured as under:
$a = 757.64 \pm 0.045$ m, $b = 946.70 \pm 0.055$ m, and $C = 54°18' \pm 25''$.
Compute the area of the triangle and its standard error.

3

MEASUREMENT OF HORIZONTAL DISTANCES

3.1 GENERAL

The determination of the distance between the given points on or above the surface of the earth, is one of the basic operations of surveying. For mapping purposes all the measured distances are finally reduced to their equivalent horizontal distances except those which are measured to determine the differences in level of the points. This may be achieved either by evolving measurement procedures to produce the horizontal distance or by computing the horizontal distance from a measured slope distance. The measurement of vertical distances is discussed in Chapter 6 on levelling.

3.2 METHODS OF DISTANCE MEASUREMENTS

The methods of making linear measurements can be classified as below:

1. Direct methods
2. Indirect methods
3. E.D.M. methods.

Before the above methods are discussed in detail, it would be better to have some idea of approximate methods of measuring the distances, given below. These methods may be used in reconnaissance, or for detecting major mistakes.

Pacing: A distance between two points can approximately be determined by counting the number of paces, and multiplying it with average length of the pace.

Passometer: It is a small instrument which counts the number of paces.

Pedometer: This instrument directly gives the distance by multiplying the number of paces with the average pace length of the person carrying the instrument.

Odometer: It is a simple device which can be attached to the wheel of a bicycle or any such vehicle. It measures the distance by counting the number of revolutions made by the wheel to which it is attached, and multiplying the number of revolutions with the perimeter of the wheel.

Direct Methods of Distance Measurements

The direct distance measurement in the field is done using *chain* or *tape,* and the process of measuring the distance is known as *chaining* when a chain is used and *taping* when a tape is used. Formerly, for the survey of ordinary precision, the engineer's chain or Gunter's chain was used to measure the length of a line. For measurements of the highest precision special bars or tapes were used. The engineer's chain is 100 ft long and has 100 links, each 1 ft long. At every 10 links brass tags are fastened. Notches on the tags indicate the number of 10 link segments between the tag and the end of the tape. The

Gunter's chain, also known as surveyor's chain, is 66 ft long and divided into 100 links, each link of 0.66 ft. In chaining, the distances are recorded in chains and links.

The precision of distance measured with tapes depends upon the degree of refinement with which the measurements are taken. It is possible to get a relative precision better than 1/1,000,000 in distance measurements using tapes by taking extreme care to eliminate all possible errors.

Indirect Methods of Distance Measurements

In the indirect methods, the distances are not directly measured in the field. The distances are computed indirectly using other observed quantities. Such methods are discussed in Chapter 8 on tacheometry.

EDM Method of Distance Measurements

The electromagnetic or electronic distance measuring instruments referred to as EDM's are practically replacing the measurement of distances using tapes. There is a large variety of such instruments, and depending upon the precision required, a particular EDM instrument should be employed. The EDM instruments and their principles of working are discussed in Chapter 11 of *Higher Surveying*.

3.3 EQUIPMENT FOR DISTANCE MEASUREMENTS

The direct distance measurements are made using measuring tapes. The tapes are made of a variety of material, length, and weight, and some of those which are commonly used in surveying, are briefly discussed below.

Linen tape: Linen tape is a painted and varnished strip of woven linen about 15 mm wide, attached to a spindle in a leather case into which it is wound when not in use (Fig. 3.1). These tapes are light and handy but not very accurate as they are subject to serious variations in length.

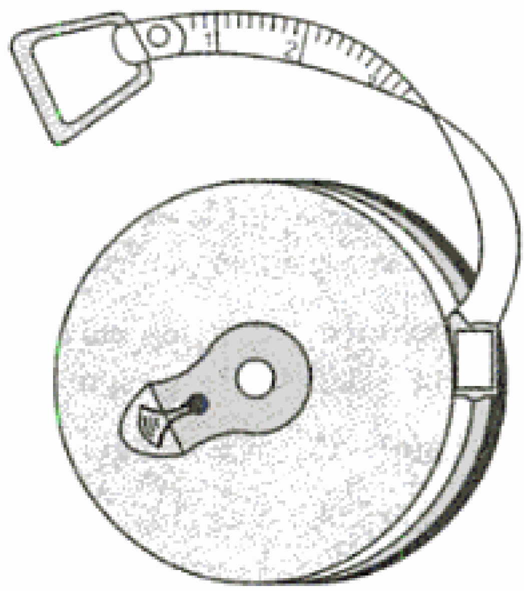


Fig. 3.1 Tape

Metallic tapes: These are woven ribbons of water proof fabric into which are woven thin brass or bronze wires to prevent stretching. Like linen tapes, these are also attached to a spindle in a leather, plastic, or metal case. The metallic tapes are 10, 20, 30, or 50 m long. Nonmetallic glass fibre tapes which are quite flexible, strong and nonconductive, are also available, and are used in the vicinity of power lines or electrical equipment.

Invar tapes: For high precision measurements such as those for base lines, the tapes made of the alloy *invar*, is used. It has a very low coefficient of thermal expansion, about 1.2×10^{-7} per degree centigrade.

Other Equipment for Distance Measurement

For direct measurement of long lines of more than a tape length, the following instruments are used (Fig. 3.2):

(*i*) Plumb bob

(*ii*) Ranging rods and poles

(*iii*) Pegs
(*iv*) Arrows or taping pins
(*v*) Spirit level
(*vi*) Spring balance
(*vii*) Tape clamp
(*viii*) Line ranger.

Plumb bob: The plumb bob is a pointed metal weight used to locate the horizontal position of a point above the ground.

Ranging rods and poles: These may be metallic, wooden, or of fibreglass. They are used as temporary signals to indicate the location of points or the direction of lines. The common length of the ranging rods is 2 or 3 m. Usually they are painted with alternate bands of white and red or black.

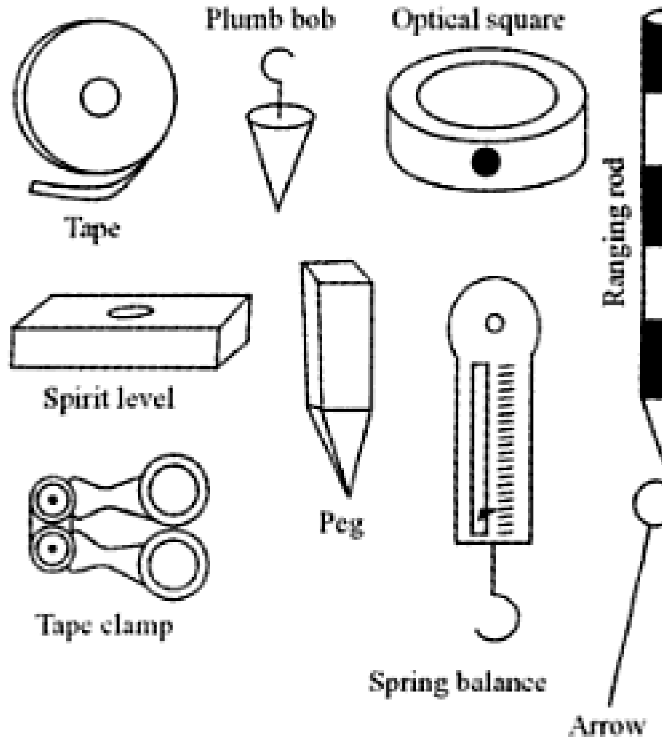


Fig. 3.2 Equipment for taping

Pegs: The wooden pegs are used to mark the end points of a survey line or positions of the survey stations. They are generally square in section, and tapered at one end. The common size of the wooden pegs is 25 mm × 25 mm × 150 mm.

Arrows or taping pins: The arrows are employed to mark the ends of the tape during the process of taping between two points which are more than a tape length apart. They are usually 25 to 35 cm long.

Spirit level: A Spirit level is employed to keep the two ends of the tape in same horizontal plane when taping over a sloping or irregular terrain. A hand level is also sometimes used.

Spring balance: A spring balance is used to apply desired pull or tension to the tape during the measurement. The tension is expressed in kilograms.

Tape clamp: The tape clamp is a device for holding a tape at any place other than at an end.

Line ranger: An optical square a kind of line ranger is an optical instrument, and is employed to establish intermediate points on a straight line joining two distant points without having to sight from one of them. The principle of operation can easily be understood from Fig. 3.3.

To range a point *P* on the line *AB*, two ranging rods are fixed at the two ends *A* and *B*, and the observer holds the line ranger at *P* very near to the line by eye judgement (Fig. 3.3a). If *P* is not on the line *AB*, the two images of the ranging rods at *A* and *B*, will not be seen in one vertical line as shown in Fig. 3.3b. The line ranger is moved back and forth along a line approximately at right angles to *AB*, until the images of *A* and *B* appear to be in one vertical line as shown in Fig. 3.3c. The point *P* is then transferred to the ground with the help of a plumb bob.

3.4 PROCEDURES FOR DISTANCE MEASUREMENTS

To measure the length of a survey line, the chain or tape has to be stretched from end to end. This is possible only when the length of the survey line is less than the tape length. However, if the length of the line is greater than the tape length, the length of the line cannot be measured accurately without establishing intermediate points on the line by the method of ranging.

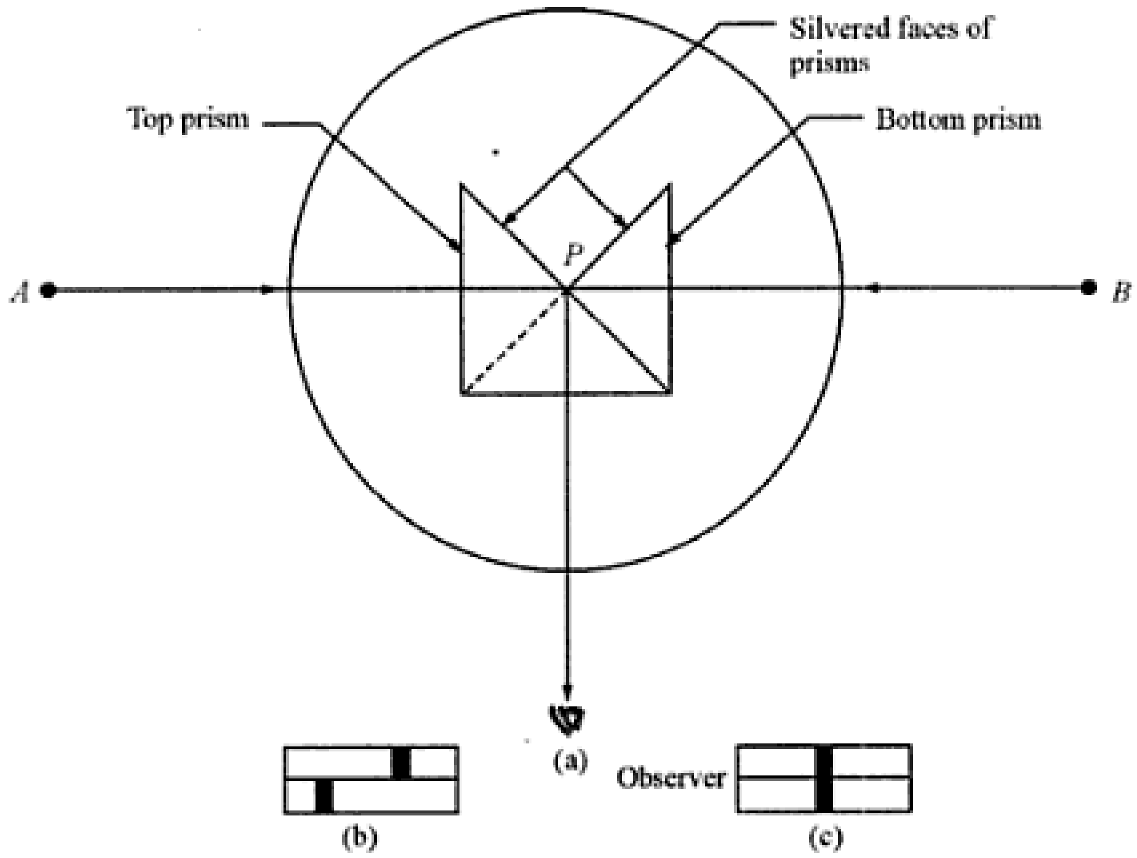


Fig. 3.3 Working principle of line ranger

Measuring a distance with chains was originally called chaining. The term is still in use and is often applied to the operation of measuring distances with tapes which is commonly termed as taping. The procedures discussed below are specific to chaining in chain surveying, and are equally applicable to taping.

Ranging

The term *ranging* is used to establish a set of intermediate points on a straight line whose two ends have already been fixed on the ground, whereas chaining means the measuring the length of a straight line with chain or tape. Chaining a long line necessarily involves ranging.

Let *A* and *B* are the two ends of a line *AB* as shown in Fig 3.4, and they are intervisible. On this line let three intermediate points *P*, *Q*, and *R* are to be established. To perform ranging for establishing the points *P*, *Q*, and *R*, the surveyor stands at a point *S*, close to the end *A* on the line *BA* produced. The assistant holds a ranging rod vertically, approximately near the line *AB*. By eye judgment the surveyor directs the assistant to move the ranging rod so that *A*, *P*, and *B* appear to be in one line. Similarly the other points *Q* and *R* are also fixed.

The procedure described above is also used to establish points 1 and 2 on either side of the ends *A* and *B* of the line.

Fig. 3.4 Ranging a line when its ends are intervisible

There may be a case when in a valley the whole length of the line is not visible from the ends of the line. In such cases, the ranging is accomplished by fixing a point A_1 near the edges of the valley as shown in Fig. 3.5. The intermediate part A_1C of the line AB is not seen from O. Therefore, the ranging is only possible for the part CB of the line but not of the part A_1C. By fixing a point A_1 at the edge of the valley, the points 1, 2, 3, etc., can be fixed on the line AB. Points 1′, 2′, 3′, etc., lying in the part A_1C are fixed by ranging from B taking help of A_1 and, the subsequently fixed points.

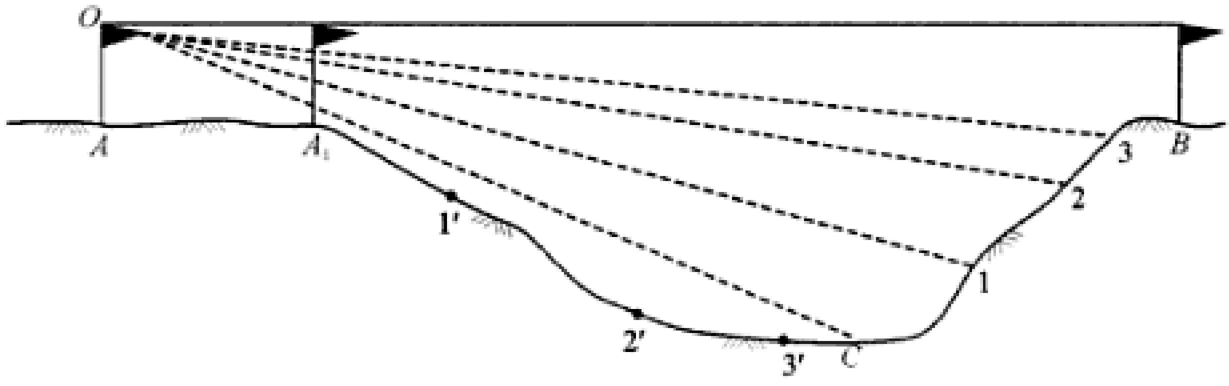


Fig. 3.5 Ranging a line in valley

In another case, the ends of the line may not be intervisible. In such cases, the method of *reciprocal ranging* is employed. In Fig. 3.6a, the ends A and B of the line AB are not intervisible due to an intervening hill.

To fix the intermediate points on the line, choose two points C and D such that D and B are visible from C and, C and A from D.

Let the initial selected position of C is C_1. Taking help of C_1 and B, range D at D_1. Then taking help of D_1 and A, range C_1 at C_2. By repeating this procedure the selected points move closer and closer to AB, and after a few repetitions, the points C and D are finally located on the line.

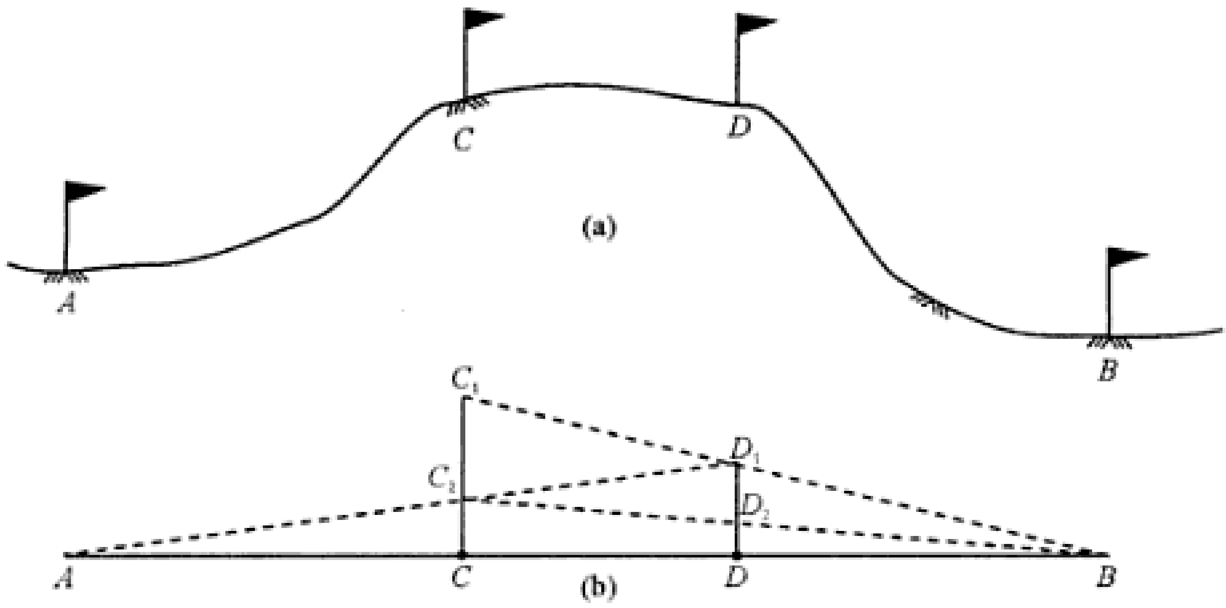


Fig. 3.6 Reciprocal ranging

Taping on a Flat Ground

The horizontal distance between the points can be measured with a tape directly when the length of the line is less than the length of the tape by laying straight the tape on the ground. If the length of the line is more than the length of the tape, the line is subdivided into parts of lengths less than the tape length. The intermediate points for the subdivision of the line, are fixed.by ranging discussed in Sec. 3.4.1. In case of a moderately flat ground for obtaining better results, both the ends of the tape are kept at the same elevation using a hand level or by estimation, and transferring the ends of the tape on the ground by plumbing as shown in Fig. 3.7. The correction for sag discussed in Sec.3.5, is applied to the measured lengths for each span to get the final length of the line.

Fig. 3.7 Horizontal distance measurement on a moderately flat ground

Taping on a Sloping or Uneven Ground

If the slope of the ground is more than 3° or 1 in 20, there is considerable difference between the slope distance and the horizontal distance. One of the following procedures may be adopted to determine the horizontal equivalent of the measured slope distance in such cases:

1. Direct method
2. Indirect method.

Direct method: In the direct method, also known as *stepping method*, the horizontal equivalent of the slope distance is directly measured, as shown in Fig. 3.8a. It is more convenient to measure downhill than to measure uphill and, therefore, uphill measurement should be avoided as far as possible. In the case of downhill measurements, the horizontal distance is measured in steps as shown in Fig. 3.8a using the procedure explained for taping on the flat ground.

For taping uphill as shown in Fig. 3.8b, the rear end of the tape is held at A' above A and the other end at B on the line AE, and the measurement proceeds upwards in similar manner as above.

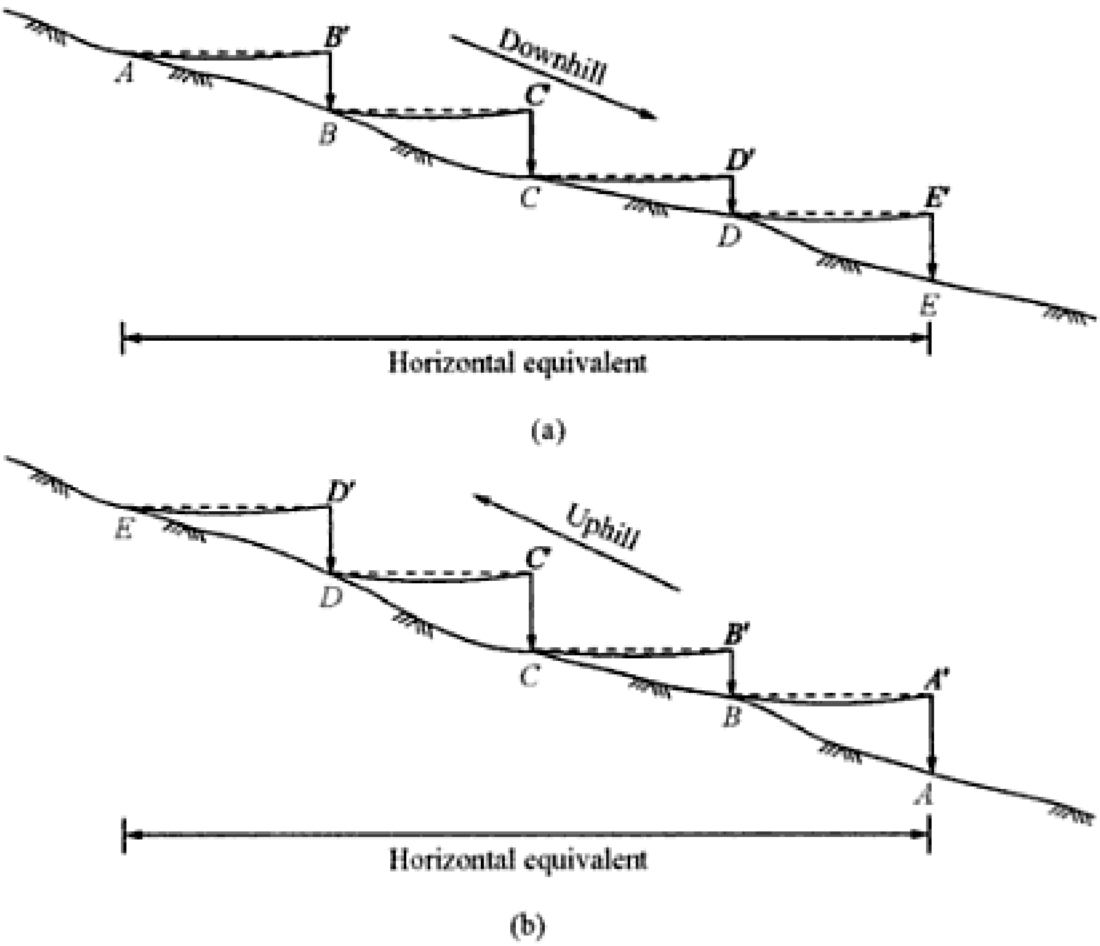


Fig. 3.8 Horizontal distance measurement on a sloping ground

Indirect method: In this method the slope distance S is measured directly with a tape, and the slope angle α shown in Fig. 3.9, is measured with some other instrument.

The horizontal equivalent is computed from

$$D = S \cos \alpha \qquad ...(3.1)$$

If any instrument for measuring the angle α is not available and the surveyor has hand level, the horizontal distance can be determined by measuring the difference in elevation h between the two ends of the line from the following expression:

$$D = \sqrt{S^2 - h^2} \qquad ...(3.2)$$

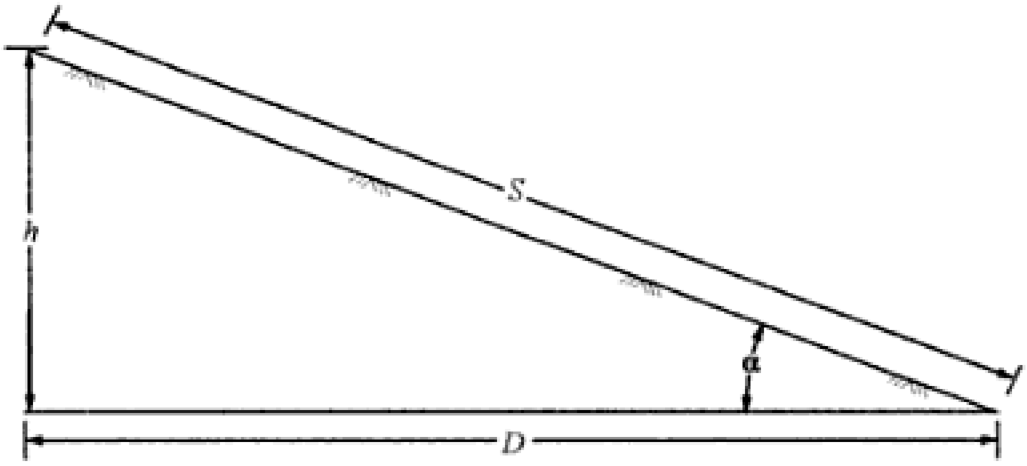


Fig. 3.9 Indirect measurement of horizontal distance

To determine the accuracy with which the vertical angle α must be measured in order to meet a given relative accuracy in the resulting horizontal distance, differentiate Eq. (3.1) with respect to α. Thus

$$dD = -S \sin \alpha \, d\alpha \qquad ...(3.3)$$

in which $d\alpha$ in radians, is the required accuracy in measurement of α. Disregarding the negative sign, the *relative accuracy* is then

$$\frac{dD}{D} = \frac{S \sin \alpha \, dx}{S \cos \alpha}$$

or

$$\frac{dD}{D} = \tan \alpha \, dx \qquad ...(3.4)$$

Further, the difference C between the slope distance and the horizontal distance can be determined by expanding the right side of Eq. (3.2) by the binomial expansion to give

$$D = S - \frac{h^2}{2S} - \frac{h^4}{8S^3} - ..... \qquad ...(3.5)$$

Since $h << S$, neglecting the terms containing higher powers of h in Eq. (3.5), we get

$$S - D = \frac{h^2}{2S} \qquad ...(3.6)$$

or
$$C = \frac{h^2}{2S} \qquad ...(3.7)$$

where $C = S - D$

Differentiating Eq. (3.7) with respect to h, we have

$$dC = \frac{hdh}{S}$$

Thus, the expression for relative accuracy is

$$\frac{dC}{S} = \frac{hdh}{S^2} \qquad ...(3.8)$$

3.5 ERRORS IN TAPING AND TAPE CORRECTIONS

The mistakes or blunders are the gross errors, and can easily be identified before any tape correction is applied for the systematic errors (*cf.* Sec. 2.4). The gross errors in taping may be due to:

1. the displacement of arrows,
2. the omission of one or more units by miscounting,
3. reading numbers incorrectly,
4. calling numbers wrongly,
5. reading from the wrong end of the tape, and
6. wrong recording of readings, etc.

The systematic errors in taping and the respective tape corrections are discussed below. The tape corrections are required to be applied depending on the field conditions under which the measurements were carried out and the requirement of precision in measurements such as in the length of base line in triangulation system.

Correction for absolute length: If absolute (actual) length of the tape is different from the designated or nominal length of the tape, the difference between the two causes a systematic error. If a tape is in continuous use, due to wear and tear, stretch or shrink, the absolute length of the tape will become different from the designated length. The correction for this difference is also called as *correction for standardization,* and is determined from the following expression:

$$c_a = \frac{C}{l} L \qquad ...(3.9)$$

where

c_a = the correction for absolute length,
C = the correction per tape length,
l = the designated or nominal length of the tape, and
L = the measured length of the line.

If the absolute length is more than the nominal length, the sign of correction is positive and *vice versa.*

Correction for temperature: The tapes are standardized in the laboratory at a certain standard temperature. If the field temperature is different from the standard temperature, a correction known as correction for temperature has to be applied to the measured length of line. The temperature correction is given by:

$$c_t = \alpha\,(t_m - t_o)\,L \qquad ...(3.10)$$

where

c_t = the correction for temperature,
α = the coefficient of linear expansion of tape material,
t_m = the mean temperature during the measurement, and
t_o = the standard temperature.

If t_m is more than t_o, the temperature correction is positive and *vice versa*. Thus, sign of c_t is same as that of $(T_m - t_o)$.

Correction for pull: The pull applied in the field during measurements, may be different from the standard pull at which the tape was standardized. This causes another correction known as correction for pull or tension, and is determined from the following expression:

$$c_p = \frac{(P - P_o)}{AE} L \qquad ...(3.11)$$

where

c_p = the pull correction,
P = the pull applied in the field during measurement,
P_o = the standard pull,
A = the cross-section area of the tape, and
E = the modulus of elasticity of the tape material.

The sign of c_p will be same as that of $(P - P_o)$.

Correction for sag: When the measurement is made by stretching the tape above the ground, supports are needed at the ends of the tape. Consequently, the tape sags under its own weight, with the maximum dip occurring at the middle of the tape (Fig. 3.10), and the tape takes the shape of a catenary. This necessitates a correction known as correction for sag. It is given by

$$c_g = \frac{1}{24}\left[\frac{W}{P}\right]^2 L \qquad ...(3.12)$$

where

c_g = the sag correction
W = the weight of the tape per span length, and
P = the pull applied during the measurement.

The sign of c_g is always negative.

If both the ends of the line are not at the same level, a further correction to c_g is required as below:

$$c_g' = c_g \cos^2\alpha \qquad ...(3.13)$$

where α is the angle of slope between the end supports.

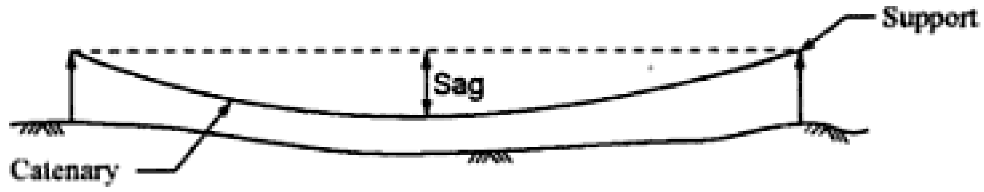


Fig. 3.10 Sagging of a tape

Correction for slope: If the two ends of the tape are at different elevations, the measured length needs a correction known as correction for slope. The expression given below is for the determination of this correction.

$$c_s = (1 - \cos\alpha)\, L \quad (exact) \qquad ...(3.14)$$

$$c_s = \frac{h^2}{2L} \quad (approximate) \qquad ...(3.15)$$

where

c_s = the slope correction,

α = the angle of the slope,

h = the difference in elevation of the two ends, and

L = the measured slope length.

c_g has always negative sign.

Correction for alignment: If the intermediate points are not in correct alignment with the ends of the line, or they are not on the line to be measured, a correction known as correction for alignment has to be applied to the measured length. It is computed from the following expression.

$$c_m = \frac{d^2}{2L} \quad (approximate) \qquad ...(3.16)$$

where

c_m = the correction for alignment, and

d = the distance by which the other end of the tape is out of alignment (Fig. 3.11).

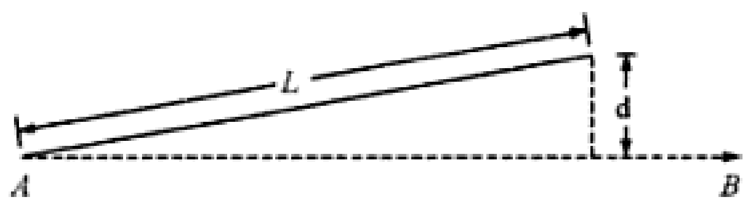


Fig. 3.11 Correction for alignment

The correction for alignment c_m is always negative.

Reduction to mean sea-level: For surveys of extensive areas, the geodetic surveys are carried out in which the base line is measured with highest precision. The measured linear distances of base lines

(after applying necessary corrections discussed above) are reduced to their equivalent sea-level lengths. This is known as reduction to mean sea-level (m.s.l.).
The reduced length at mean sea-level is given by:

$$L' = \frac{R}{R+h}L \qquad ...(3.17)$$

R = the mean radius of the earth (6370 km), and

h = the average elevation of the measured line (Fig. 3.12).

The correction to L is given by

$$c_l = \frac{hL}{R+h} \quad (exact) \qquad ...(3.18)$$

where

$$= \frac{hL}{R} \quad (approximate) \qquad ...(3.19)$$

$C_l = L - L'$.

The sign of the correction for reduction to mean sea-level is always negative.

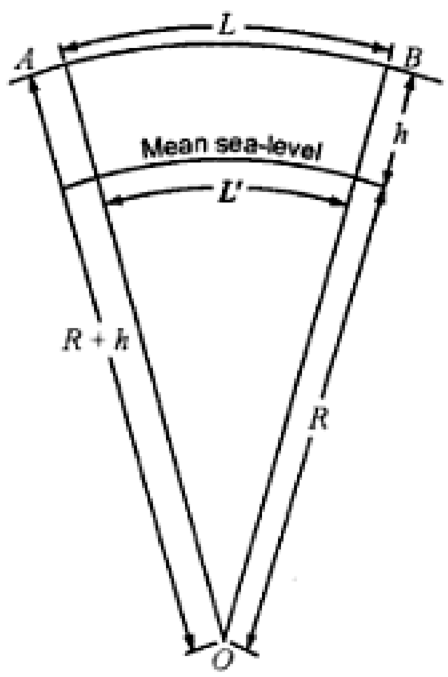


Fig. 3.12 Reduction to mean sea-level

Various tape corrections for systematic errors are summarised in Table 3.1.

Table 3.1 Corrections for systematic errors in taping

Correction	Sign	Formula
Standardisation	±	$\frac{C}{l}L$
Temperature	±	$\alpha(t_m - t_o)L$
Pull	±	$\frac{P - P_o}{AE}L$
Sag	–	$\frac{1}{24}\left(\frac{W}{P}\right)^2 L$
Slope	–	$(1 - \cos\alpha)L$ (*exact*) $\frac{h^2}{2L}$ (*approximate*)
Alignment	–	$\frac{d^2}{2L}$ (*approximate*)
Reduction to m.s.l	–	$\frac{hL}{R}$ (*approximate*)

After removal of gross errors and systematic errors, the errors which remain in the observations, are the random errors. These errors may be due to several causes. Some of these are

1. Error in determining temperature of tape
2. Failure to apply the proper tension
3. Wind deflecting the plumb bob
4. Arrows not set exactly where the plumb bob touches the ground
5. Inability of the observer to estimate the last place in reading between the `graduations.

Adjustment of random errors is discussed in Chapter 2 of *Higher Surveying*.

ILLUSTRATIVE EXAMPLES

Example 3.1

The measured slope distance between two points is 615 m. If the angle of slope between the points is 7°, what is the horizontal distance between the points?

Solution:

Slope distance $S = 615$ m

Angle of slope $\alpha = 7°$

The horizontal distance from Eq. (3.1) is

$$D = S \cos \alpha$$

$= 615 \times \cos 7°$

$= \mathbf{610.4\ m.}$

Example 3.2

The distance between two points *A* and *B*, is measured along the slope as 435 m. The difference in elevation of these points is 49 m. Determine the horizontal distance between *A* and *B*.

Solution:

Slope distance $S = 435$ m

Difference in elevation $h = 49$ m

The horizontal distance from Eq. (3.2)

$$D = \sqrt{S^2 - h^2}$$

$$= \sqrt{435^2 - 49^2}$$

$$= \mathbf{432.23\ m.}$$

Example 3.3

If the slope of the ground is 1 in 5, and the measured distance between two points is 503 m, what is the horizontal equivalent of the measured distance?

Solution:

Slope distance S = 503 m

Slope = 1 in 5

For 1 unit of vertical distance, horizontal distance is 5 units,

$$\tan \alpha = \frac{1}{5}$$

or $$\alpha = 11°18'36''$$

Thus the horizontal equivalent

$$D = S \cos \alpha$$

$$= 503 \times \cos 11°18'36''$$

$$= \mathbf{493.23\ m.}$$

Example 3.4

What slope distance must be laid out along a line that rises 5 m /100 m in order to establish a horizontal distance of 830 m?

Solution:

For 100 m of horizontal distance the vertical rise is 5 m, therefore, the vertical rise for 830 m horizontal distance would be $5 \times \frac{830}{100} = 41.5$ m.

or $\quad h = 41.5$ m

We have

$$D = \sqrt{S^2 - h^2}$$

$$S = \sqrt{D^2 + h^2}$$

$$= \sqrt{830^2 + 41.5^2}$$

$$= \mathbf{831.04 \text{ m.}}$$

Example 3.5

A measurement is made along a line that is inclined by a vertical angle of 2° 26′. The measured slope distance is 4035.46 m. To what accuracy must the slope angle be measured if the relative accuracy of the horizontal distance is to be $\frac{1}{25000}$? Also compute the horizontal distance.

Solution:

The accuracy $d\alpha$ in measurement of slope angle α, is given by Eq. (3.4) as below:

$$\tan\alpha \, d\alpha = \frac{dD}{D}$$

or

$$d\alpha = \frac{dD}{D}\frac{1}{\tan\alpha}$$

$$= \frac{1}{25000} \times \frac{1}{\tan 2°26'}$$

$$= \frac{1}{1062.3808}$$

or

$$d\alpha = \frac{206265}{1062.3808} \text{ seconds}$$

$$= 194''$$

$$= \mathbf{3' \, 14''}.$$

The horizontal distance

$$D = 4035.46 \times \cos 2°26'$$

$$= \mathbf{4031.82 \text{ m}}.$$

Example 3.6

To what accuracy must the slope angle of Example 3.5, be measured if the horizontal distance is to be accurate to 0.005 m?

Solution:

From Eq. (3.3), we have

$$S \sin\alpha \, d\alpha = dD \text{ (neglecting the sign)}$$

$$d\alpha = \frac{dD}{S \sin \alpha}$$

or
$$d\alpha = \frac{206265\, dD}{S \sin \alpha} \text{ seconds}$$

$$= \frac{206265 \times 0.005}{4035.46 \times \sin 2°26'}$$

$$= \mathbf{6.0''}.$$

Example 3.7

What is the correct length of a line which is measured as 350 m with a 20 m tape, 10 cm too long?

Solution:

From Eq. (3.9), the correction required to the measured length is

$$c_a = \frac{C}{l} L$$

$$= \frac{0.10}{20} \times 350$$

$$= 1.75 \text{ m}$$

As the tape is too long by 10 cm, the sign of the correction is positive.

Thus the correct length will be

$$= 350 + 1.75$$

$$= \mathbf{351.75\ m}.$$

Example 3.8

At the end of a survey of a parcel of land, a tape of 30 m length was found to be 10 cm short. The area of the plan drawn with the measurements taken with this tape is found to be 135 cm^2. If the scale of the plan is 1/1000, what is the true area of the field assuming that the chain was exact 30 m at commencement of the survey?

Solution:

Scale of the plan is $\frac{1}{1000}$

or 1 cm = 10 m

Measured area of the field = plan area $\times 10^2$

$= 135 \times 10^2$

$= 13500\ m^2$

If the incorrect length of the tape is L', and the designated length is L then the true area A is given by

$$A = \left(\frac{L'}{L}\right)^2 \times \text{measured area}$$

The incorrect length L' is obtained by

$$= \frac{1}{2} \text{ Length of the tape at the (commencement + end) of the survey}$$

$$= \frac{29.90 + 30.00}{2}$$

$$= 29.95 \text{ m}$$

$$A = \left(\frac{29.95}{30}\right)^2 \times 13500$$

$$= \mathbf{13455.04\ m^2}.$$

Example 3.9

A line was measured with a steel tape which was exactly 30 m at 25° C at a pull of 10 kg, the measured length being 1700.00 m. The temperature during measurement was 34°C and the pull applied was 18 kg. Compute the length of the line if the cross-sectional area of the tape is 0.025 cm^2. Take $\alpha = 3.5 \times 10^{-6}$ per 1°C and $E = 2.1 \times 10^6$ kg/cm^2 for the material of the tape.

Solution:

We have

l	=	30 m	L	=	1700 m
t_o	=	25° C	t_m	=	34° C
P	=	18 kg	P_o	=	10 kg
A	=	0.025 cm^2	α	=	3.5×10^{-6} per 1° C
E	=	2.1×10^6 kg/cm^2			

(*i*) From Eq. (3.10), the correction for temperature is

$$c_t = \alpha (t_m - t_o) L$$

$$= 3.5 \times 10^{-6} \times (34 - 25) \times 1700.00$$

$$= 0.0536 \text{ m (+ve)}$$

(*ii*) The correction for pull from Eq. (3.11) is

$$c_p = \frac{(P - P_o)}{AE} L$$

$$= \frac{(18 - 10)}{0.025 \times 2.1 \times 10^6} \times 1700.00$$

$$= 0.2591 \text{ m (+ve)}$$

The total correction $= 0.0536 + 0.2591$

$$= 0.3127 \text{ m}$$

The corrected length of the line

$$= c_t + c_p$$

$$= 1700.00 + 0.3127$$

$$= \mathbf{1700.31 \text{ m}}.$$

Example 3.10

A steel tape of nominal length 30 m was suspended between supports to measure the length of a line. The measured length of the line on a slope of angle 3°50′ is 29.859 m. The mean temperature during the measurement was 12°C and the pull applied was 100 N. If standard length of the tape is 30.005 m at 20° C, and the standard pull is 45.0 N, calculate the corrected horizontal length. Take the weight of the tape = 0.15 N/m, its cross-sectional area = 2.5 mm², $\alpha = 1.15 \times 10^{-5}$ per °C, and $E = 2.0 \times 10^5$ N/mm².

Solution:

(*i*) Correction for standardization [from Eq. (3.9)]

$$c_a = \frac{C}{l} L$$

$$= \frac{(30.005 - 30)}{30} \times 29.859$$

$$= \mathbf{+ 0.00498 \text{ m}}.$$

(*ii*) Correction for temperature [from Eq. (3.10)]

$$c_t = \alpha \, (t_m - t_o) \, L$$

$$= 1.15 \times 10^{-5} \, (12 - 20) \times 29.859$$

$$= \mathbf{- 0.00275 \text{ m}}.$$

(*iii*) Correction for pull [from Eq. (3.11)]

$$c_p = \frac{(P - P_o)}{AE} L$$

$$= \frac{(100 - 45)}{2.5 \times 2.0 \times 10^5} \times 29.859$$

$$= \mathbf{0.00328\ m.}$$

(*iv*) Correction for sag [from Eq. (3.12)]

$$c_g = \frac{1}{24}\left[\frac{W}{P}\right]^2 L$$

$$W = 0.15 \times 29.859$$

$$= 4.48 \text{ N}$$

$$c_g = \frac{1}{24}\left[\frac{4.48}{100}\right]^2 \times 29.859$$

$$= 0.0025 \text{ m}$$

$$= -\ 0.0025 \text{ m (always –ve)}$$

Further, as the supports are not at the same level [from Eq. (3.13)]

$$c_g' = c_g \cos^2\alpha$$

$$= -\ 0.0025 \times \cos^2 3°50'$$

$$= \mathbf{-\ 0.00249\ m.}$$

(*v*) The correction for slope [from Eq. (3.14)]

$$c_s = (1 - \cos\alpha)\ L$$

$$= (1 - \cos 3°50') \times 29.859$$

$$= 0.0668 \text{ m}$$

$$= -\ 0.0668 \text{ (always –ve)}$$

Total correction = 0.00498 – 0.00275 + 0.00328 – 0.00249 – 0.0668

= – 0.06378 m

Correct horizontal distance = 29.859 – 0.06378

= **29.795 m.**

PROBLEMS

3.1 Explain why the distance measured on the surface of the earth is required to be reduced to equivalent horizontal distance.

3.2 Discuss the various methods of horizontal distance measurements.

3.3 Describe different kinds of tapes commonly used in surveying stating the advantages of each. Also discuss the equipment used for distance measurements.

3.4 Differentiate between Ranging and Taping.

3.5 Describe how you would range a survey line between two points which are not intervisible.

3.6 Discuss various sources of errors in linear measurements and identify that the resulting errors belong to which category of error.

3.7 What are different tape corrections and how are they applied?

3.8 A surveyor paces a 50 m length eight times with the following results: 57, $56\frac{1}{2}$, 56, 58, $56\frac{1}{3}$, 57, $56\frac{1}{2}$, and 56 paces. How many paces must he step off in order to lay off a distance of 400 m.

3.9 The distance measured between two points along a sloping ground with a slope angle of 5°15′ is 2023.47 m. What is the horizontal distance between the points?

3.10 If the measured slope distance between two points with difference in elevation of 11.326 m, is 256.74 m, what is the horizontal distance?

3.11 The measured slope distance between two points is 336.79 m, and the angle of slope is 8°28′. Compute the difference in elevation of the two points.

3.12 A horizontal distance of 950 m is to be laid out on the slope for which the slope angle is 4°58′. What slope distance should be laid out?

3.13 In Problem (3.9), if the vertical angle is in error by 3′30″ what error is produced in the computed horizontal distance?

3.14 In Problem (3.9), if it was desired that the error in the computed horizontal distance should not be more than 0.01 m, to what an accuracy the slope angle should be measured?

3.15 A line AB was measured in segments along sloping ground with a 30 m tape, and the following measurements were recorded:

Slope distance (m)	30.00	30.00	18.92	11.46	30.00	8.23
Difference in elevation (m)	1.58	0.95	0.90	0.56	3.01	0.69

Calculate the horizontal distance of the line *AB*.

3.16 A rectangular parcel of land of 315 m × 750 m is required to be established. A 30 m chain was calibrated and found to measure 30.03 m. What measurements must be laid off on the ground?

3.17 With what accuracy must a difference in elevation between two ends of 30 m tape be known if the difference in elevation is 3.150 m, and the accuracy ratio is to be at least 1:25000?

3.18 A base line was measured with a 30 m long steel tape at 15°C, and with a pull of 100 N (10 kgf). What is the correction per tape length if the temperature at the time of measurement was 22°C and the pull exerted was 150 N(15 kgf)? The weight of the steel is 0.0768 N per 1 cm^3, the weight of the tape is 8 N, the modulus of elasticity of the tape material is 2.1×10^7 N/cm^2 and its coefficient of linear expansion is 7.0×10^7 per °C.

3.19 The length of a steel tape was found exactly 30 m at a temperature of 30° C under pull of 5 kg when lying on the flat platform. The tape is stretched over two supports between which the measured distance is 30.000 m. There are two additional supports in between equally spaced. All the supports are at the same level, and the tape is allowed to sag freely between the supports.

Calculate the actual length between the two outer supports, and the reduced length at mean sea-level, if the mean temperature during the measurement was 37 °C, and the pull applied was 9 kg. The average elevation of the terrain is 945 m.

$$
\begin{aligned}
\text{Weight of the tape} &= 1.50 \text{ kg} \\
\text{Area of cross-section of the tape} &= 6.5 \text{ mm}^2 \\
\alpha &= 1.2 \times 10^{-5} \text{ per °C} \\
E &= 2.1 \times 10^6 \text{ kg/cm}^2 \\
R &= 6372 \text{ km}
\end{aligned}
$$

3.20 A base line *AB* was measured in two parts *AC* and *CB* of lengths 1540 m and 1919 m with a steel tape which was exactly 30 m at 20 °C at a pull of 10 kg. The applied pull during measurement of both parts was 25 kg whereas the respective temperatures were 40°C and 45°C. The slopes of the ground for the two parts were +2°40′ and +3°10′, and the deflection angle of *CB* was 11ñ right. Find the correct length of the base line. The cross-section area of the tape is 0.025 cm², the coefficient of linear expansion and modulus of elasticity of tape material are 2.5×10^{-6} per °C and 2.1×10^6 kg/cm², respectively.

4

MEASUREMENT OF ANGLES AND DIRECTIONS

4.1 GENERAL

The purpose of a survey is to determine the relative locations of points on, below or above the surface of the earth. Since the earth is three-dimensional, a three-dimensional rectangular coordinate system, $x'y'z'$, called the *geocentric coordinate system (right handed)* shown in Fig. 4.1, is required to locate the points. For geodetic surveys, points are located using spherical coordinates consisting of latitude ϕ, longitude λ, and the distance $(R + h)$ along the normal to the earth ellipsoid.

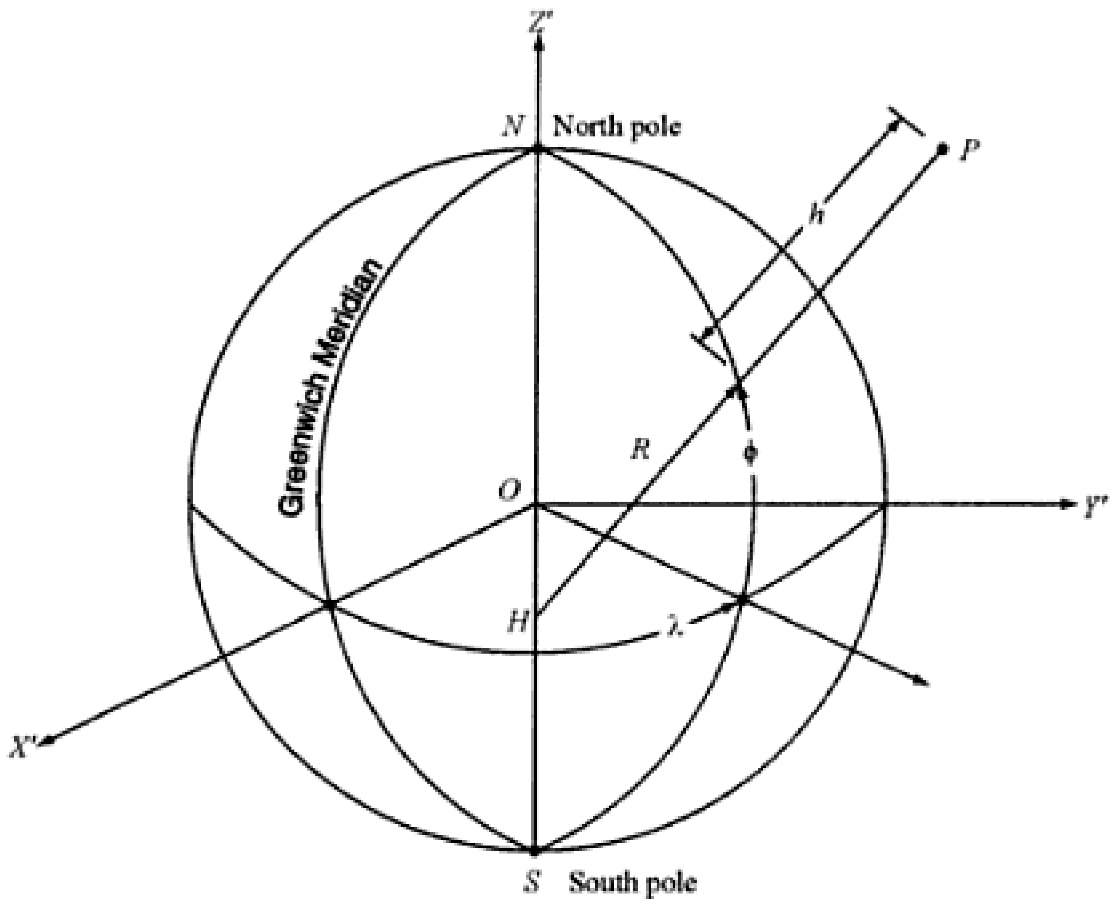


Fig. 4.1 Geocentric coordinate system

For plane surveying, a more convenient local xyz coordinate system as illustrated in Fig. 4.2, is adopted. The origin of such a system is usually taken near the centre of the area to be surveyed, z-axis coincides with the local vertical or the plumb line, and xy-plane is a horizontal plane (i.e., xy-plane is tangent to the reference ellipsoid at the point of origin). The y-axis is generally taken towards the North pole. To locate any point P, either the coordinates (x,y,z) of the point or the two angles θ in horizontal plane known as horizontal angle, and α in vertical plane known as vertical angle, may be used, and the distance r of P from the origin O.

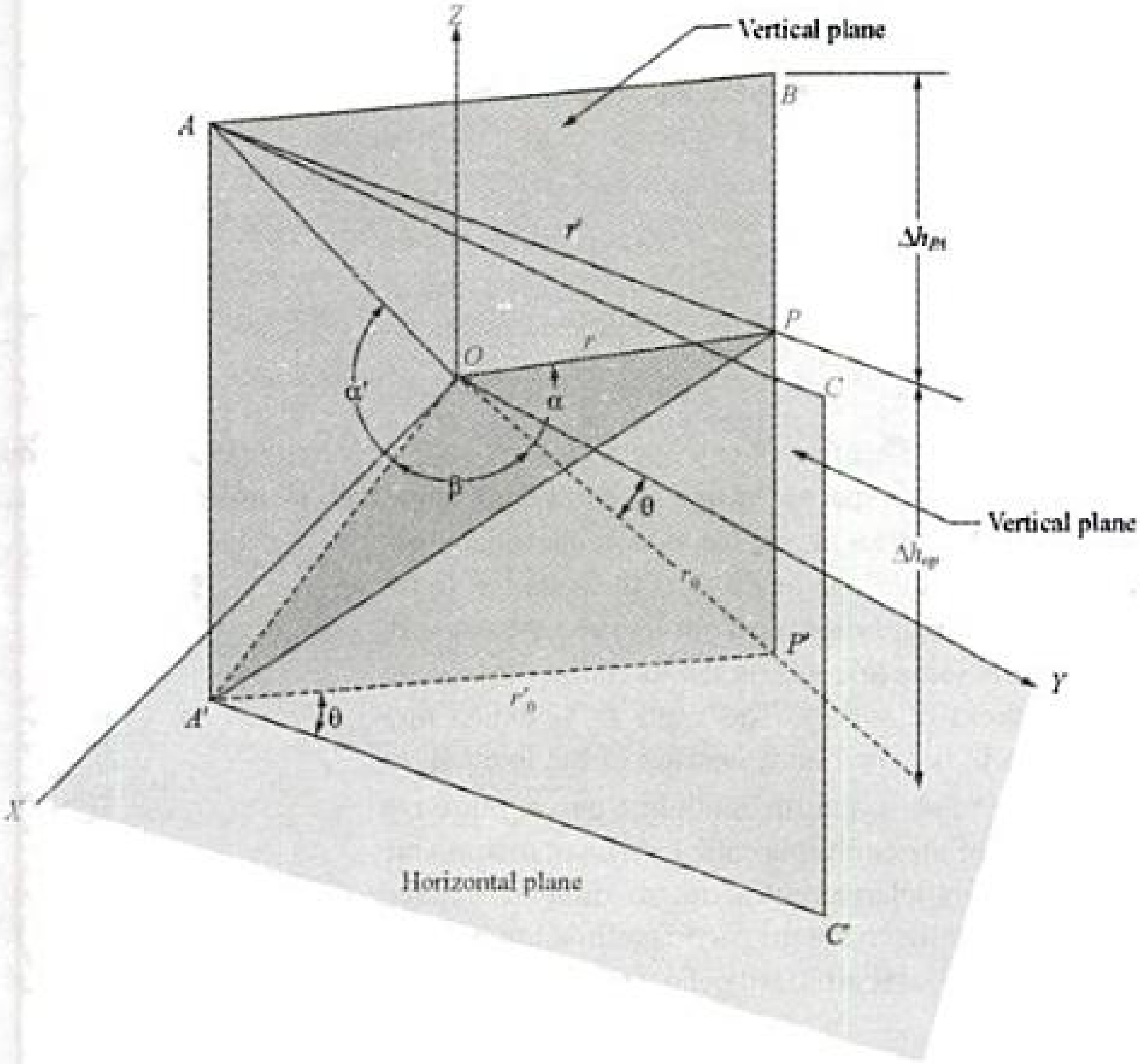


Fig. 4.2 Local Coordinate System

In surveys of small extent, the position of the points P and A, and their slope distances r and r', are projected onto the horizontal plane xy as P', A', r_0, and r_0', respectively. Thus, any appropriate combination of the following measurements may be used to locate the point in space:

1. Horizontal angle measurements
2. Vertical angle measurements
3. Horizontal distance measurements
4. Vertical distance measurements.

The projection of the points on the horizontal plane may be defined by horizontal angles alone, horizontal distances alone, or horizontal angle and horizontal distance. For example, P' can be defined in the horizontal plane by one of the following:

1. Distance r_0 and direction θ from one known point O.
2. Directions θ and θ' from two known points O and A'.
3. Distances r_0 and r_0' from two known points O and A'.

For any two points P and A, the horizontal angle between them at O is the angle β, measured in the horizontal plane xy. The vertical angle to a point is the angle of elevation if the point is above the horizontal plane, or angle of depression if the point is below the horizontal plane, always measured from the horizontal plane. Thus the vertical angle to P at O is α and to point A is α'. The height difference between the points O and P is Δh_{OP}, and that of the points P and A is Δh_{PA} as shown in the figure.

4.2 ANGLES AND DIRECTIONS

There are several means of defining the angles and directions. These are:

1. Bearings
2. Azimuths
3. Deflection angles
4. Angles to the right
5. Interior angles.

Bearings

Bearing is defined as the direction of any line with respect to a given meridian. A fixed reference line *OM*, shown in Fig. 4.3, with respect to which the horizontal angles are measured in clockwise direction, is known as *meridian*. Any straight line has two diametrically opposite directions. For example, the angle α_1 is measured at *O* for the line *OA* in the direction of progress of survey, and the angle β_1 is measured at *A* for the same line *AO*, in the direction opposite to the direction of progress of survey. The angle α_1 is called *fore bearing* and the angle β_1 is called *back bearing* of the line *OA*.

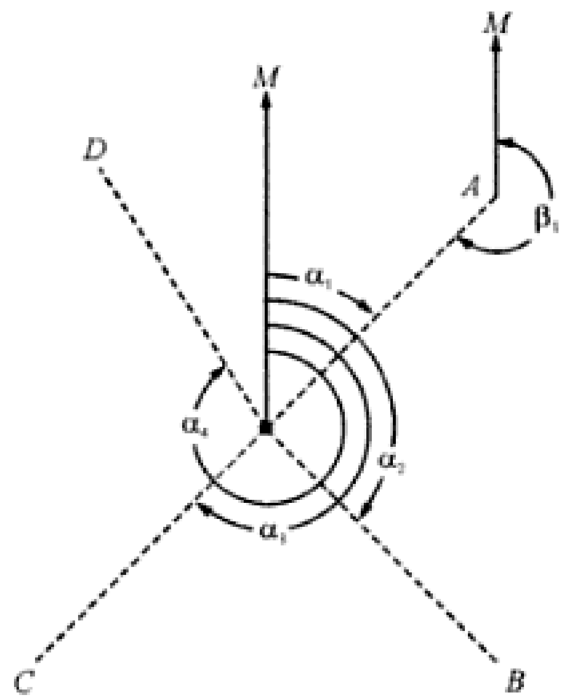
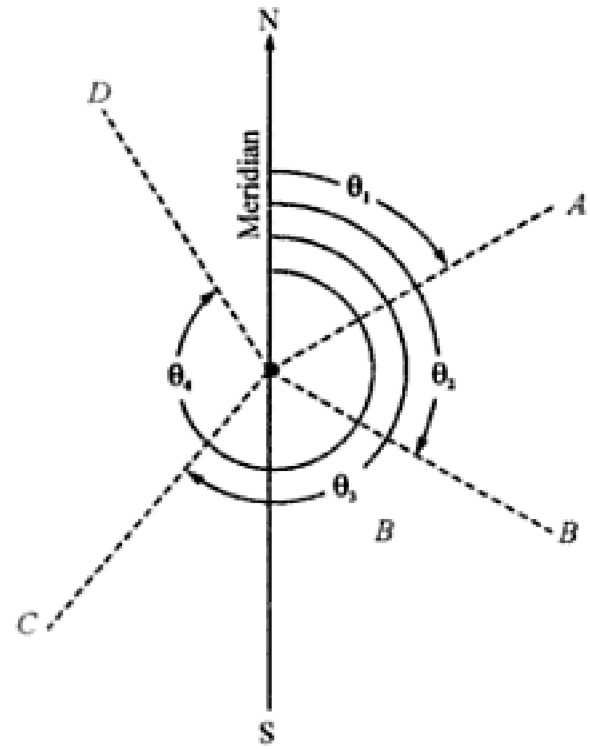


Fig. 4.3 Directions referred to a meridian

If the fixed reference line is a north-south line passing through the geographical poles of the earth it is called a *true* or *astronomic meridian*, if it is a line parallel to a central true meridian it is called a *grid meridian* if it is parallel to the magnetic north-south line it is a *magnetic meridian*, and if it is arbitrarily chosen it is an *arbitrary* or *assumed meridian*.

As the bearings depend on the meridian chosen they may be *true bearings, magnetic bearings* or *assumed bearings*.

If the bearings θ_1, θ_2, θ_3, etc., are measured clockwise from north side of the meridian as shown in Fig. 4.4a, they are *whole-circle bearings*. The acute angle between the reference meridian and the line is known as the *reduced bearing* or *quadrantal bearing*. If the bearings are measured from north branch of the meridian in clockwise direction and the line falls in the 0° to 90° quadrant as shown in Fig. 4.4b, it is written as NθE. If the line lies in the 270° to 360° quadrant, the bearing is measured from the north branch of meridian, and is written as NθW. Similarly, if the line lies in 90° to 180° quadrant and 180° to 270° quadrant, the bearings are measured from the south branch of the meridian, and are written as SθE and SθW, respectively. In Fig. 4.4b, the reduced bearings of the lines *OA*, *OB*, *OC*, and *OD* are Nθ_1E, Sθ_3E, Sθ_4W, and Nθ_2W, respectively.



(a)
Fig. 4.4 (a)

Azimuths

The angle between a line and the meridian measured in clockwise direction usually from the north branch of the meridian is the *azimuth* of the line. Also, some surveyors reckon azimuths from

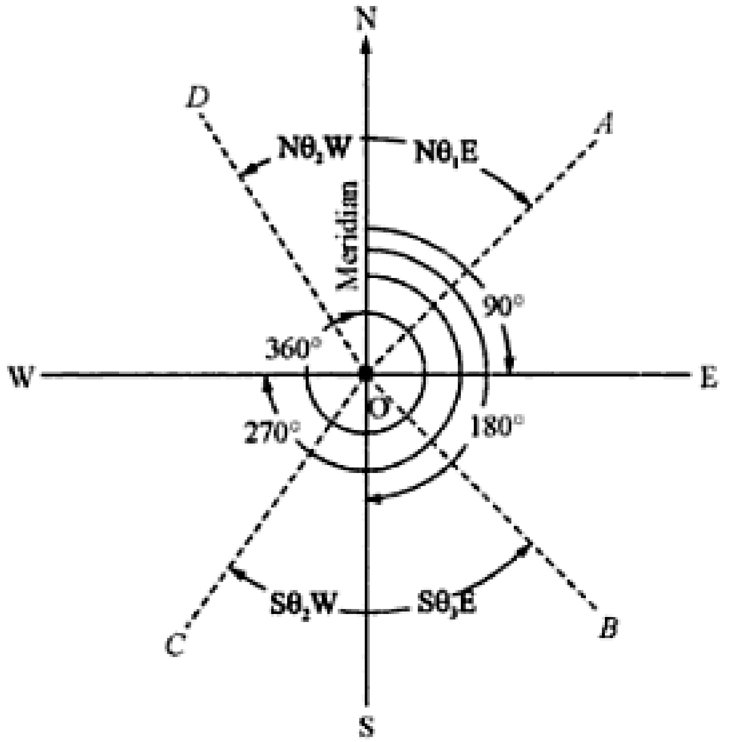


Fig. 4.4 (b) Whole-circle bearing and quadrantal or reduced bearing

the south branch of the meridian, and therefore, it becomes necessary to specify that the azimuth is from the north or from the south. Azimuths may be true (astronomic), magnetic, or assumed depending on the reference meridian adopted. They have values ranging from 0° to 360°. Thus, the azimuths are in fact whole-circle bearings.The two may be distinguished from each other when the convergence of meridians is considered in determination of azimuth. In practice the term azimuth is used in geodetic and astronomical surveying whereas the term bearing is used in plane surveying. If the direction of a line *OA* is defined from an origin *O* to the terminal point *A* as shown in Fig. 4.3, the azimuth is a *forward azimuth*. Conversely, the azimuth from *A* to *O* is the *back azimuth* of *OA*. The concept of forward and back azimuth of a line are only valid in plane surveying where there is no convergence of meridian.

Deflection Angles

The angle between a line and the prolongation of the preceding line is known as a *deflection angle*. The deflection angle may be on the right or left of the prolongation, and they are recorded as right or left depending on the angle measured on the right or left as shown in Fig. 4.5.

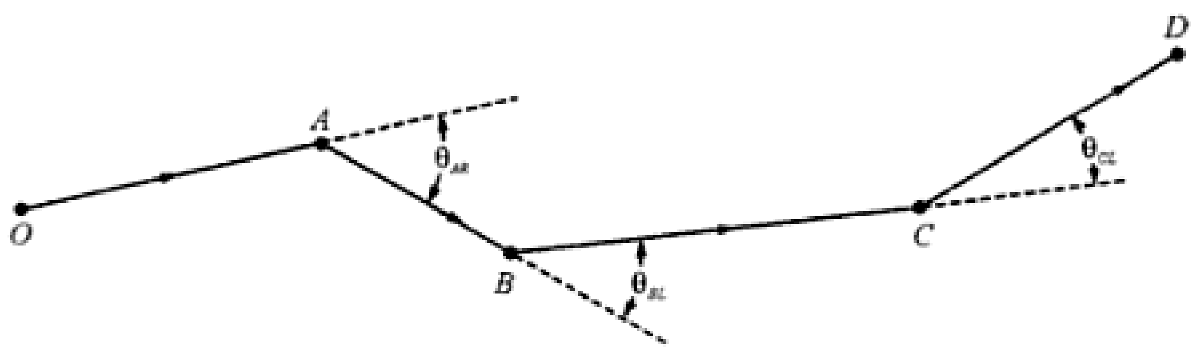


Fig. 4.5 Deflection angles

Angles to the Right

In a survey if the angles are measured clockwise from the preceding line to the following line, as illustrated in Fig. 4.6, the measured angles are called *angles to the right*.

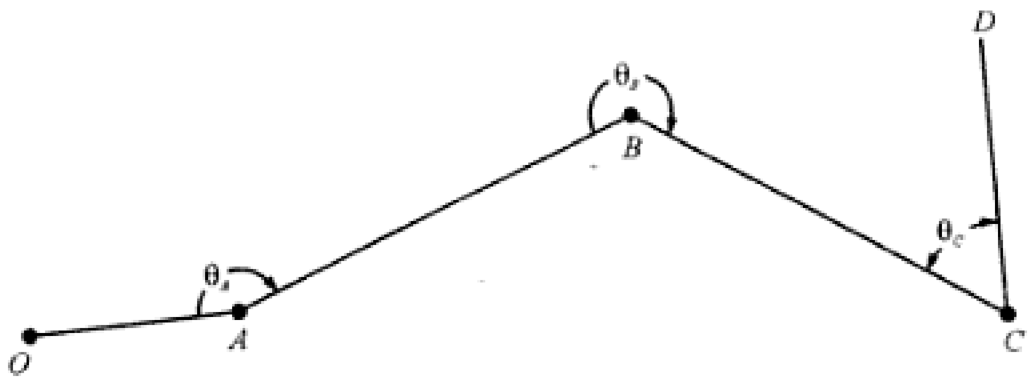


Fig. 4.6 Angles to the right

Interior Angles

The inside angles between adjacent lines of a closed polygon are called *interior angles*. The exterior angles, shown by dashed lines in Fig. 4.7, are the explements of interior angles.

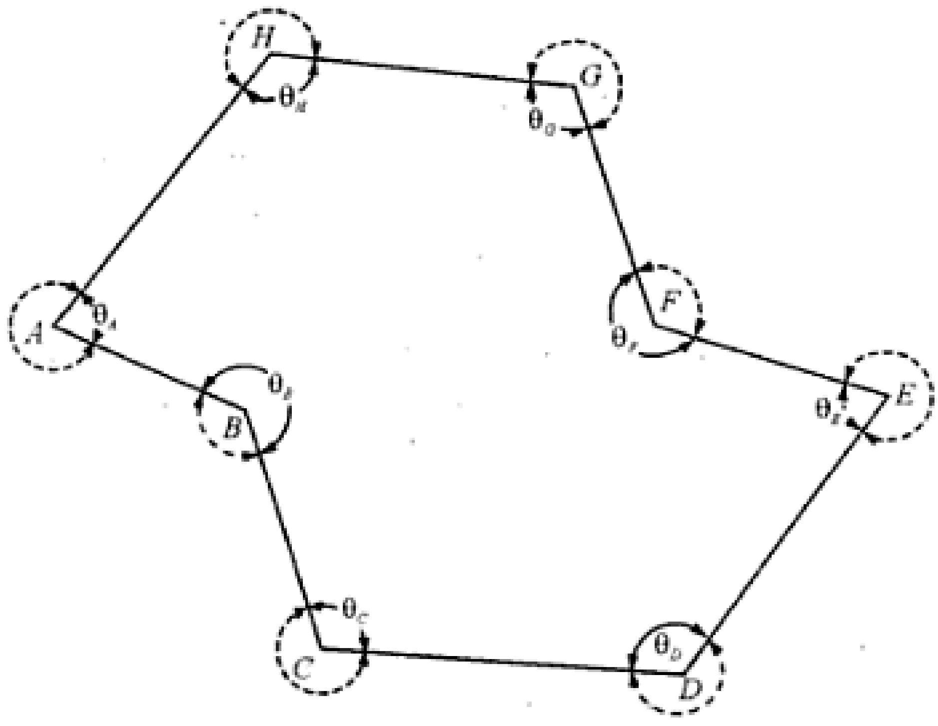


Fig. 4.7 Interior angles

4.3 METHODS OF DETERMINING ANGLES AND DIRECTIONS

The method of determining angles and directions depends on the instrument employed. Normally a transit theodolite or simply theodolite is the most commonly used instrument for measuring angles but can also be determined by means of a tape, plane table and alidade, compass, or sextant. Directions are determined with a direction theodolite, or with a magnetic compass.

In the following sections description of the instruments and methods of measurement have been discussed.

4.4 THEODOLITE

A theodolite shown in Fig. 4.8, is an instrument designed to measure horizontal and vertical angles. The sectional view of a theodolite given in Fig. 4.9 shows its main parts.

Parts of a Theodolite

A theodolite consists of the following essential parts.

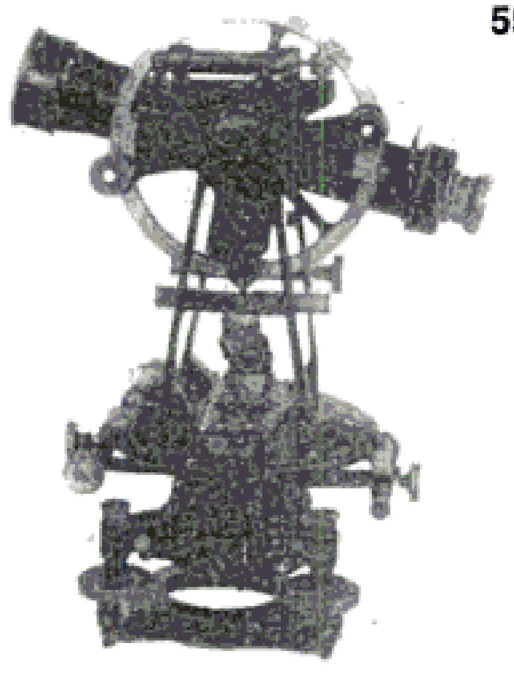


Fig. 4.8 Theodolite

Levelling head: It consists of upper tribrach and lower tribrach. The upper tribrach has three arms, and each arm carries a levelling screw for supporting and levelling the instrument. The boss of the upper tribrach is pierced with a female axis in which the lower male vertical axis operates.

The lower tribrach has a circular hole through which a plumb bob may be suspended for centering the instrument over the ground mark.

The functions of a levelling head are to support the main part of the instrument, to attach the theodolite to the tripod, and to provide a means for levelling the theodolite.

Lower plate: The lower plate of the instrument, also known as vernier plate, is attached to the outer spindle and carries a graduated circle at its bevelled edge. The graduated circle is divided into 360° and each degree is further subdivided in 10′ or 20′ of arc intervals. The plate can be clamped at any desired position by a lower clamp screw and a corresponding slow motion can be made with a tangential screw or slow motion screw.

Upper plate: The upper plate is attached to the inner spindle axis. Two diametrically opposite verniers, named as *A* and *B*, fitted with magnifiers, are fixed to the upper plate. The upper plate carries two standards to support a telescope, and one or two level tubes to level the instrument. It also carries an upper clamp screw and a corresponding tangent screw or slow motion screw. On clamping the upper clamp and unclamping the lower clamp, the instrument may be rotated on its outer spindle without any relative motion between the two plates, and thus causing no change in graduated circle reading. On the other hand, if the lower clamp is tightened and the upper clamp is unclamped, the instrument may be rotated on its inner spindle with a relative motion between the upper plate and lower plate. This property is utilized for measuring the angles between two settings of the instrument.

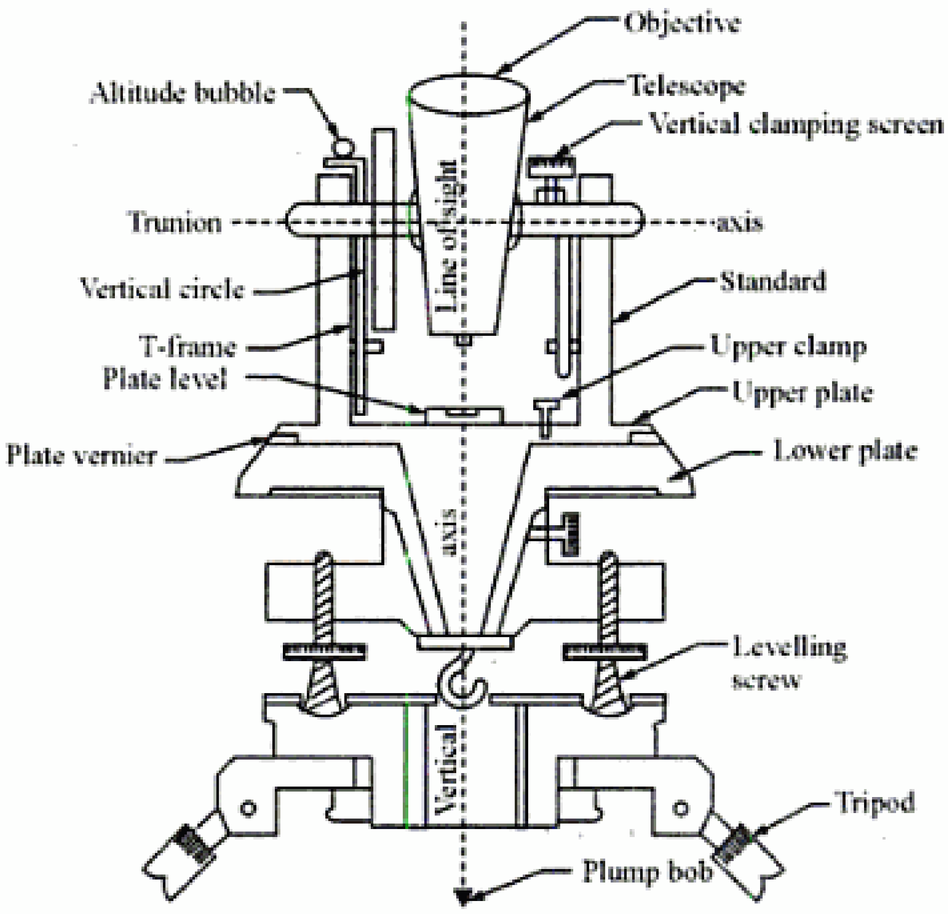


Fig. 4.9 Sectional view of a theodolite

Standards (or A Frame): Two standards resembling the English letter *A*, are firmly attached to the upper plate. The tops of these standards form the bearing of the pivots of the telescope allowing it to rotate on its trunion axis in vertical plane. The T–frame and arm of the vertical circle clamp are also attached to the standards.

T–frame or Index bar: T–frame resembles the English letter T, and is centered on the horizontal axis of the telescope in the frame of the vertical circle. The two verniers, named as *C* and *D*, are provided on it at the ends of the horizontal arm, called the index arm. A vertical leg known as clipping arm, is provided with a fork and two clipping screws at its lower extremity. At the top of this frame is attached a bubble tube called the *altitude bubble tube*.

Plate levels: The upper plate carries one or two plate levels. In the case of two plate levels, they are placed at right angles to each other. The plate level bubbles can be centered with the help of the foot screws.

Telescope: The telescope of a theodolite may be the internal focussing type or the external focussing type. The internal focussing telescopes are the most commonly used telescopes because of the following advantages:

1. The overall length of the telescope is not altered during focussing and hence the balance of the telescope is not affected.
2. There is no risk of breaking the plate levels while transiting the telescope.
3. The line of collimation is least affected by focussing.
4. The telescope remains free from dust and moisture.
5. Wear of rack and pinion is less due to lesser movement of the concave lense.
6. By fitting a concave lens, the diameter of the objective can be increased without aberrative effects.
7. The combined focal length of the lenses, increases the power of the telescope.
8. In tacheometric observations, the additive constant is generally eliminated (*cf.*, Chapter 8).

The telescope being one of the most essential components of a theodolite and another surveying instrument level, a detailed knowledge of a telescope is helpful not only to regular users but also in adjustment of instrument.

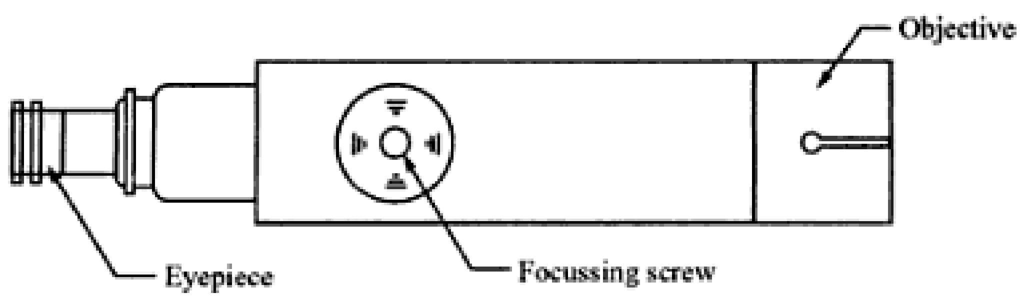


Fig. 4.10 Telescope

A telescope has two main parts, an *eyepiece*, held next to the eye and the *objective*, towards the object sighted (Fig. 4.10).

The objective or object glass of a surveying telescope consists of a compound lens, an outer double-convex lens and an inner convexo-concave lens, the two being cemented together at their common surface (Fig. 4.11)). The eyepiece used in a surveying telescope is of the Ramsden type shown in Fig. 4.12. It has two plano-convex lenses of equal focal length, mounted with their convex surfaces facing each other, and separated by a distance equal to two-thirds of the focal length of either lens.

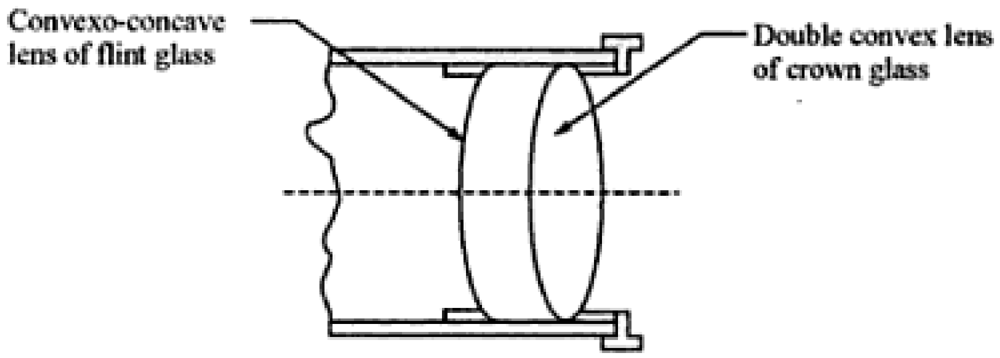


Fig. 4.11 Objective of a telescope

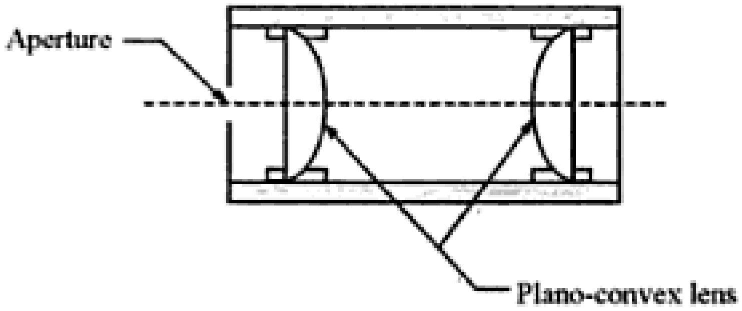
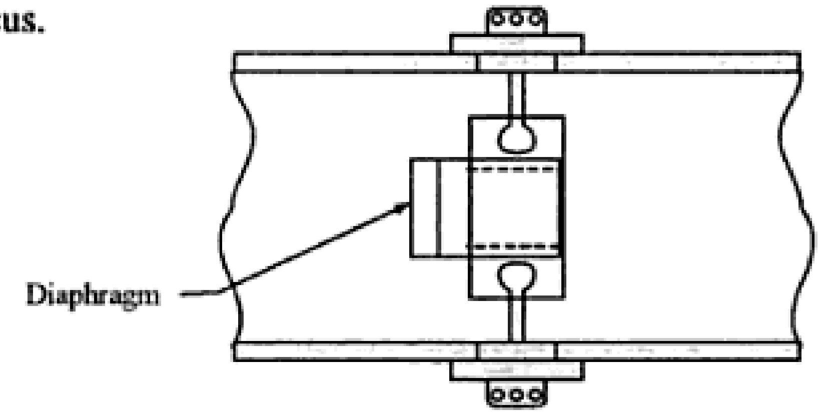
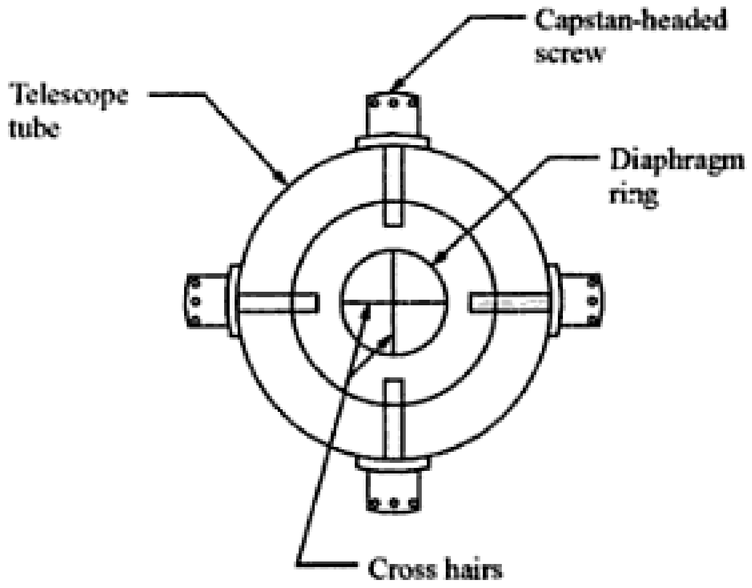


Fig. 4.12 Eyepiece of a telescope

Both the objective and the eyepiece are set in cylindrical metal cases of different sizes, that of the eyepiece being smaller, one sliding axially inside the other with a rack and pinion arrangement attached to a focussing screw. The focussing screw moves the objective relative to the eyepiece in order to bring the object into clear focus.



(a) Longitudinal section



(b) Cross section

Fig. 4.13 Diaphragm of a telescope

The *diaphragm* containing the cross hairs is another important part of the telescope. It is placed between the eyepiece and objective but nearer the former. This is held in place in the telescope tube by means of four capstan-headed screws as shown in Fig. 4.13. The capstan-headed screws allow the horizontal and vertical adjustments of the cross hairs.

Spider-web cross hairs are the simplest and commonly used with metal rings or glass diaphragms (Fig. 4.14 a). The cross hair patterns shown in Fig. 4.14a and b, are common in levels, and the other patterns are for theodolites and tacheometers.

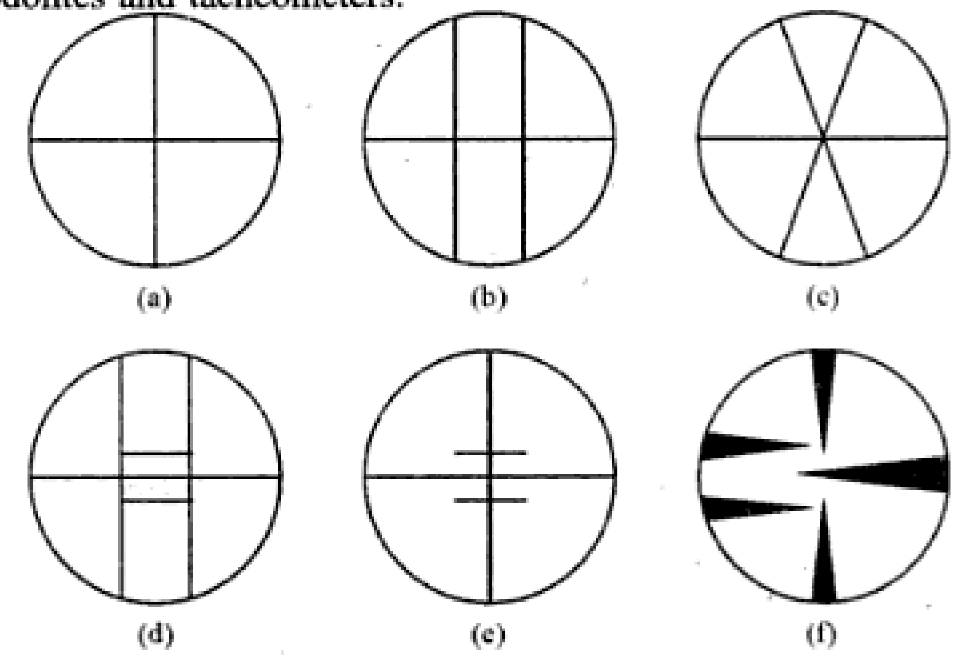


Fig. 4.14 Patterns of cross hairs

Since the primary purpose of a diaphragm is to facilitate sighting or striking an object, the image of the object must be brought into focus in the plane of the cross hairs. The line passing through the optical centre of the objective and entering the observer's eye through the eyepiece, is called the *line of sight*. The imaginary line joining the optical centre of the objective and the intersection of the cross hairs, is called the *line of collimation*.

Vertical circle: A vertical circle is attached with the telescope and is graduated in various ways by the manufacturers. The following graduations of vertical circles are in common use:

1. The vertical circle is divided into four quadrants each reading from 0° to 90°. The 0° - 0° line is a vertical line (Fig. 4.15a).
2. The vertical circle is divided into four quadrants from 0° to 90° in both the directions, 0° being at the eyepiece end and objective end (Fig. 4.15b). The 0° - 0° line is a horizontal line.

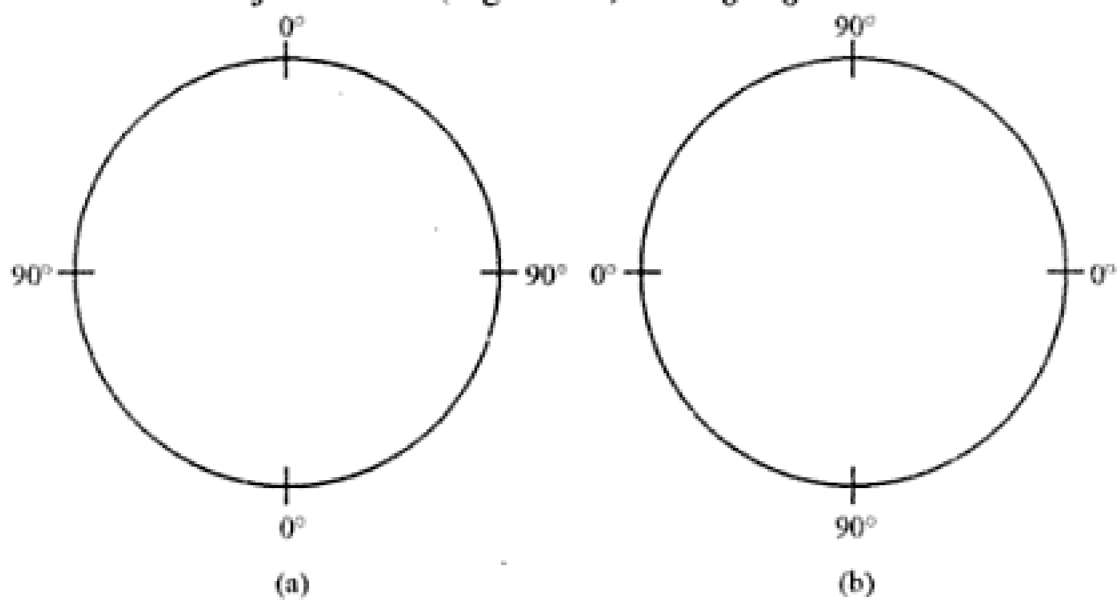


Fig 4 15 Graduation systems of vertical circle

Tripod: The theodolite is supported on a tripod consisting of three solid or framed legs for taking the observations. The lower end of each leg has pointed iron shoe for providing greater stability to the instrument while working. The tripod head carries at its upper surface an external screw to which foot plate of the levelling head is screwed.

Plumb bob: To centre the instrument exactly over the ground mark, a plumb bob is suspended from the hook fitted to the bottom of the main vertical axis.

Definitions and Other Technical Terms

The following terms are associated with the angle measurements with a theodolite (Fig. 4.16):

Vertical axis: The axis (B) about which a theodolite is rotated in a horizontal plane is the vertical axis of the instrument. About this axis both upper and lower plates rotate.

Horizontal or trunnion axis: The axis (C) about which the telescope along with the vertical circle, rotates in vertical plane is called horizontal or trunnion axis.

Line of collimation: It has already been defined in Sec. 4.4.1. It is also known as line of sight (D).

Axis of the plate level bubble: The straight line (A) tangential to longitudinal curve of the level tube at its centre, is the axis of the plate level bubble (Fig. 4.17).

Centering: The process of setting up a theodolite over the ground station mark, is known as centering. When a theodolite is centered over the station mark, the vertical axis of the theodolite passes through the station mark.

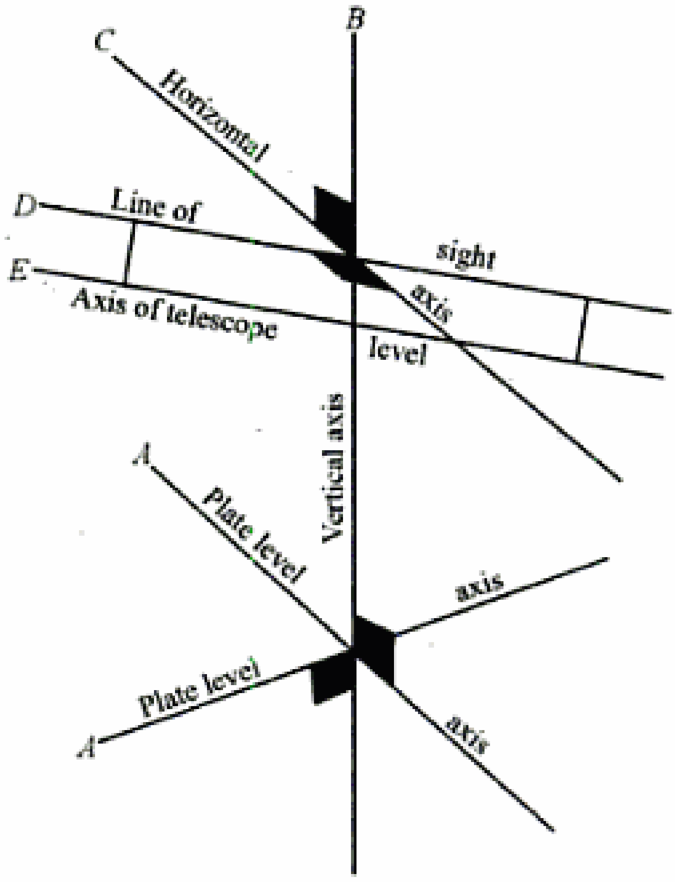


Fig. 4.16 Geometry of a theodolite

Transiting: The process of turning the telescope in vertical plane through 180° about its horizontal axis is known as transiting. The terms *reversing* and *plunging* are also used sometimes for transiting.

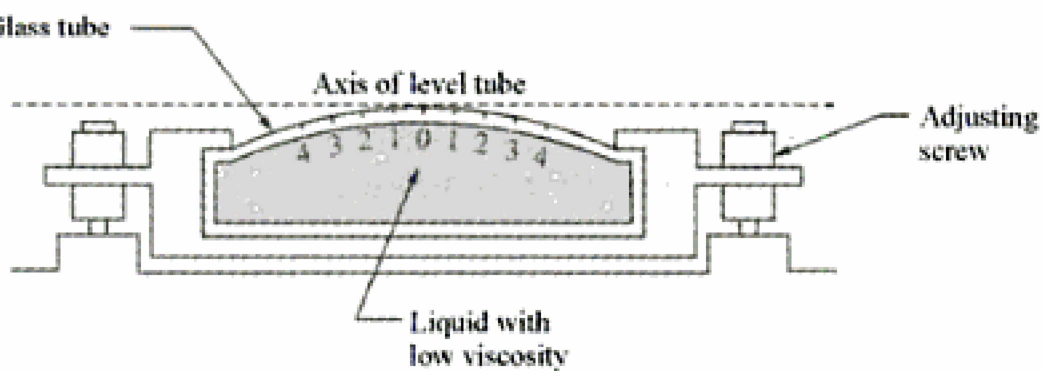


Fig. 4.17 Level tube

Swing: The continuous motion of the telescope about the vertical axis in horizontal plane is called swing. When the telescope is rotated clockwise, it is *right swing* and if anticlockwise it is *left swing*.

Face left and right observation: The observations made keeping the vertical circle on the left of the telescope are known as face left observations, and if the vertical circle is kept on the right of the telescope, the observations are called face right observations.

Changing face: When the face of the telescope is changed from left to right or *vice versa*, the process is known as changing face.

Telescope normal: When the vertical circle is on the left of the telescope and bubble is up, the telescope is said to be normal.

Telescope inverted: A telescope is said to be inverted or reversed when its vertical circle is to its right and the bubble of the telescope is down.

A set: A set of horizontal observations consists of two horizontal observations, one on the face left and the other on the face right.

Geometry of the Theodolite

A knowledge of the geometry of the theodolite is necessary for a thorough understanding of its operation. Fig. 4.16 shows the geometry of the theodolite in schematic form. In a perfectly constructed and adjusted instrument, the following geometrical requirements of relationships between the axes of the instrument, should exist:

1. The vertical axis (B) is perpendicular to the plane of the plate level bubble (A).
2. The horizontal axis (C) is perpendicular to the vertical axis (B).
3. The line of sight (D) is perpendicular to the horizontal axis (C).
4. The axis (E) of the level tube attached to the telescope is parallel to the line of sight (D).
5. The vertical axis (B), the horizontal axis (C) and the line of sight (D) pass through a single point known as *instrumental centre*.

The above relationships are achieved by permanent adjustment procedures detailed in the following section.

Adjustments of a Theodolite

The adjustments of a theodolite are of two kinds:

1. Temporary adjustments
2. Permanent adjustments.

Temporary adjustments

Theodolite must be adjusted at every set-up of the instrument before making observations. These adjustments are therefore temporary. They include the following:

1. Setting up the instrument over the station
2. Levelling of the instrument
3. Elimination of the parallax.

Setting up: The operation of setting up includes the centering of the theodolite over the ground station mark, and approximate levelling of the instrument.

The centering is carried out in following steps:

1. Suspend the plumb bob with a string attached to the hook fitted to the bottom of the vertical axis.
2. Place the theodolite over the station mark by spreading the legs of the tripod, well apart so that the telescope is at a convenient height.
3. Do approximate centering by moving the legs radially and circumferentially till the plumb bob hangs within 1 cm horizontally of the station mark.
4. Do finer centering by unclamping the centre-shifting arrangement.

Before centering the instrument over the station mark it should be ensured that the level of the tripod head is approximately levelled. In case there is a considerable dislevelment, the centering will be disturbed when levelling is done. The approximate levelling may be done with reference to a small circular bubble attached to the tribrach, or by eye judgement.

Levelling of theodolite: The levelling operation is performed to make the vertical axis of the instrument truly vertical, and pass through the ground station mark. The following steps are involved in the levelling operation (Fig. 4.18):

1. Turn the horizontal plate until the longitudinal axis of the plate level becomes parallel to the line joining any two levelling screws (Fig. 4.18a).
2. By turning the levelling screws simultaneously in opposite directions either inwards or outwards, bring the bubble to the centre of its run.
3. Turn the instrument through $180°$ in azimuth.
4. If the bubble position is found different, move it by means of the same levelling screws to the approximate mean of the two positions.
5. Turn the instrument through $90°$ in azimuth so that the plate level becomes perpendicular to the previous position (Fig.4.18b).
6. Using the third levelling screw move the bubble to the approximate mean position already indicated.
7. Repeat the above steps until the bubble retains the same position for every setting of the instrument in azimuth.

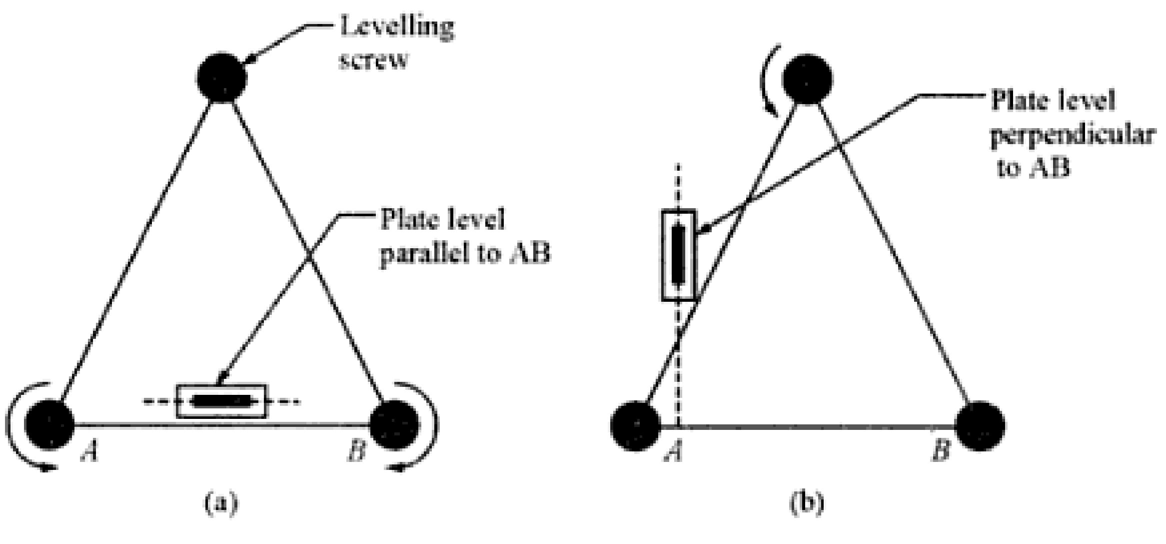


Fig. 4.18 Levelling a theodolite

Elimination of parallax: For making an observation first focus the eyepiece for the observer's eye. This is most easily done by holding a white paper about 15 cm in front of the objective and turning eyepiece in or out until the cross hairs are sharp and distinct. Then an object is selected and focused, and

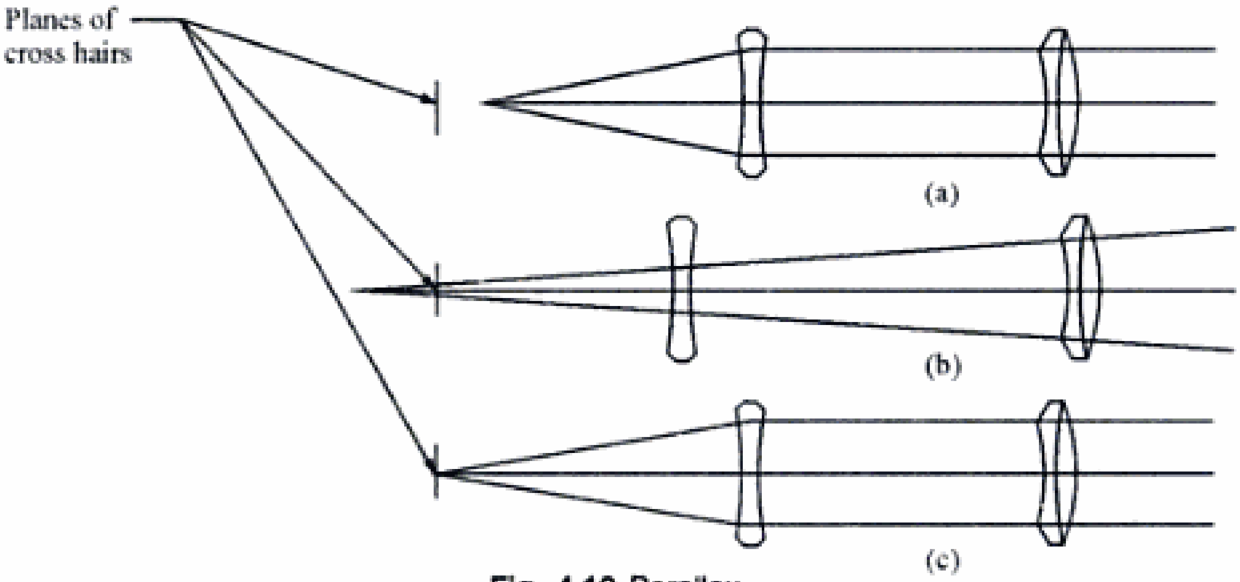


Fig. 4.19 Parallax

the image of the object is brought into the plane of the cross hairs. This can be checked by moving the eye up and down or sideways a small amount to detect whether or not the cross hairs appear to move with respect to the object sighted. If there is no relative movement, the focussing lens is in the proper position and the telescope is properly focused (Fig. 4.19c). If there is relative motion between the cross hairs and the object sighted, the condition called *parallax* exists between the image and the cross hairs. This condition arises when the focussing lens is not in its proper position as shown in Fig.4.19a and b, and the image is not formed in the plane of cross hairs. Refocussing brings the focussing lens in the proper position, and the parallax is eliminated.

Permanent adjustments

The permanent adjustments need to be made once in a while or before embarking on a precise job. They are based on the geometry of the theodolite discussed in Sec. 4.4.3.

It is possible that while carrying out some adjustments these may upset other adjustments. To avoid such possibility, the permanent adjustment should be made in the following order:

1. Make the vertical cross hair to lie in a plane perpendicular to the horizontal axis.
2. Make the plate bubble central to its run when the ver.ical axis is truly vertical.
3. Make the line of sight perpendicular to the horizontal axis.
4. Make the horizontal axis perpendicular to the vertical axis.
5. Make the telescope level bubble central when the line of sight is horizontal.
6. Make the vertical circle indicate zero when the line of sight is perpendicular to the vertical axis.

To make the vertical cross hair lie in a plane perpendicular to the horizontal axis

This adjustment is tested as below:

1. Sight the vertical cross hair on a well-defined point about 60 m away from the instrument.
2. Clamp both the horizontal motions of the instrument. Swing the telescope through a small vertical angle, so that the point traverses the length of the vertical cross hair.

3. If the point does not appear to move continuously on the hair and departs from the cross hair, adjust the vertical hair by the following procedure:

   (*a*) Loosen the two adjacent capstan-headed screws, and rotate the cross hair ring in the telescope tube until the point traverses the entire length of the hair.

   (*b*) Tighten the two capstan-headed screws.

The object of this adjustment is to bring the horizontal hair of the eyepiece into the horizontal plane through the optical axis.

The manufacturers generally take care that the vertical and horizontal cross hairs are perpendicular to each other. Therefore, if the vertical cross hair is set perpendicular to the horizontal axis, the horizontal cross hair is automatically made horizontal, and the horizontal hair of the eyepiece lies in the horizontal plane through the optical axis of the instrument and, therefore, no adjustment is required for the horizontal hair.

Adjustment of plate level

This adjustment makes the axis of the plate level perpendicular to the vertical axis of the instrument. When the plate level is in perfect adjustment, its bubble must remain at the centre of its run during complete revolution of the theodolite in azimuth.

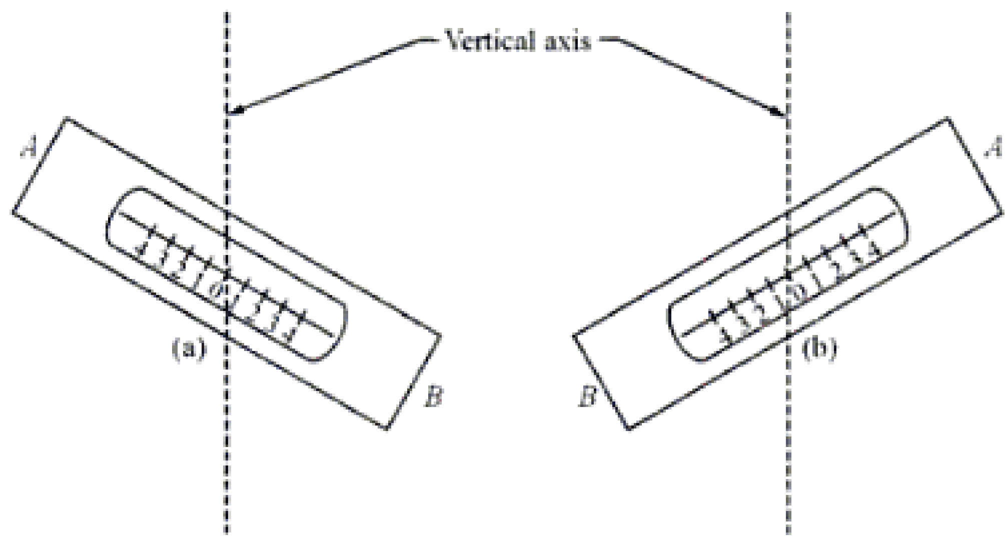


Fig. 4.20 Two positions of plate level

To test whether the plate bubble remains at its centre of run in all positions on a complete revolution of the instrument in azimuth or not, the following operations are required:

1. Set up the instrument on a firm ground. Clamp the lower plate, and turn the upper plate until plate level becomes parallel to any pair of levelling foot screws. Bring the bubble to the centre of its run by means of levelling screws as explained in Sec. 4.4.4.
2. Rotate the instrument about the vertical axis through 180°. The plate level is now again parallel to the same pair of levelling screws but the ends reversed in direction as shown in Fig. 4.20. If the bubble remains central, the vertical axis of the instrument is perpendicular to the plate level axis.

If the plate level bubble does not remain central, the adjustment of the plate level axis is done in the following steps:

1. Note down the reading of the bubble which is the apparent error and is twice the actual error of the axis of the plate level.

2. Correct half the error by means of the same pair of levelling screws.
3. Remaining half of the correction is made by means of the capstan-headed screw provided at the end of the level tube.

Repeat the test and adjustment until the bubble remains central at its run during full revolution of the instrument in azimuth.

Adjustment of vertical hair

This adjustment is to make the line of collimation perpendicular to the horizontal axis of the instrument. If this adjustment exists, the line of collimation will generate a plane when the telescope is revolved in vertical plane, otherwise a cone is generated with the horizontal axis as its axis.

To find out that whether the line of collimation is perpendicular to the horizontal axis, the following test is carried out:

1. Level the instrument and sight a point *P* at about 100 m away from the instrument position *O*, with telescope normal. *P* should be nearly at same elevation as instrument axis (Fig. 4.21).

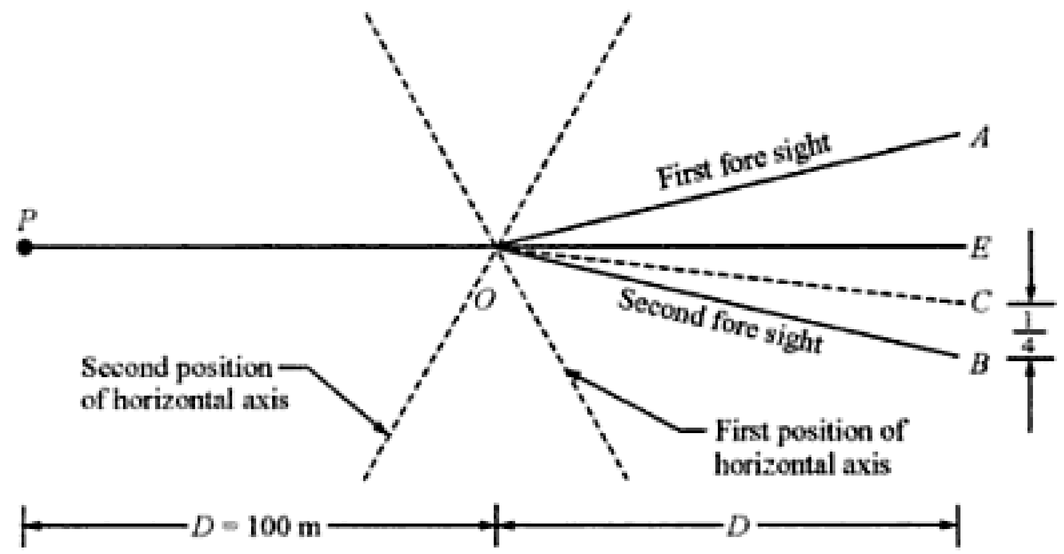


Fig. 4.21 Adjustment of vertical hair

2. Clamp both the plates. Transit the telescope, and fix *A* on the line of sight at the same distance away from *O* as *P*.
3. Unclamp the upper plate, swing the telescope and again sight *P*. The telescope is in inverted position.
4. Clamp the upper plate and transit the telescope. If the line of sight passes through *A*, the line of sight is perpendicular to the horizontal axis.

If the line of sight does not pass through *A* in step 4 above, the adjustment is done as follows:

1. Set a point *B* on the line of sight beside *A*.
2. Mark a point *C* at one-fourth distance of *AB* from *B*.
3. Adjust the vertical hair by means of two opposite capstan headed screws so that the line of sight passes through *C*.
4. Test the adjustment again and if needed, repeat the adjustment.

Adjustment of horizontal axis

With this adjustment the horizontal axis is made perpendicular to the vertical axis. This adjustment insures that the line of sight revolves in a vertical plane perpendicular to the horizontal axis when the instrument is levelled. This adjustment is very necessary for prolonging of straight lines by making observations on one face only.

The adjustment of the horizontal axis by the *spire test* is shown in Fig. 4.22. To test the adjustment the following steps are required:

1. Set up the instrument near any tall object on which a well-defined point *A* is available at about 60° to 70° vertical angle.
2. Level the instrument to make the vertical axis truly vertical.
3. Sight *A* keeping both upper and lower plates clamped, depress the telescope and fix a point *B* on the ground.
4. Transit the telescope and swing the telescope through 180° until positions of two standards are interchanged.
5. Bisect *A* again and depress the telescope and note that whether the line of sight passes through *B* or not. If it passes, the horizontal axis is perpendicular to the vertical axis and if not, the instrument needs adjustment.

To adjust the horizontal axis the procedure to be followed is as under:

1. When the line of sight does not pass through *B*, fix a point *C* on the line of sight beside *B*.
2. Locate *M* halfway between *B* and *C*. *M* will lie in the same vertical plane which contains *A*.
3. Sight on *M*, elevate the telescope until the line of sight is beside *A*.
4. Loosen the screws of the bearing cap, and raise or lower the adjustable end of the horizontal axis until the line of sight is in the same vertical plane with *A*.

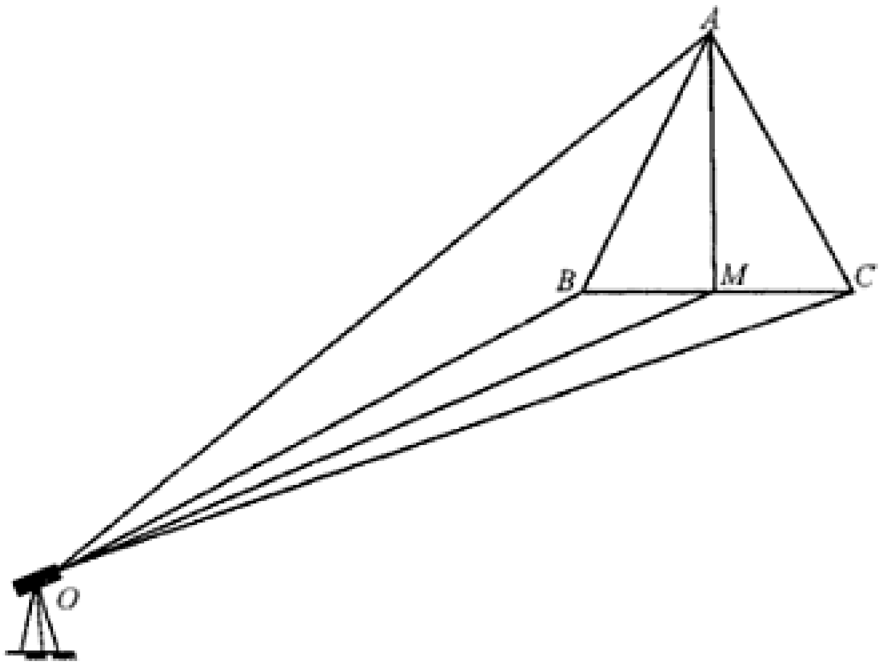


Fig. 4.22 Adjustment of horizontal axis

To make the axis of the telescope level parallel to the line of sight

This adjustment ensures that the line of collimation should remain horizontal when the bubble of the telescope level is at the centre of its run.

To test whether this adjustment exists, follow the steps as under (Fig. 4.23):

1. Set two pages A and B on fairly level ground about 100 m apart.
2. Set up the instrument at M on the line AB and equally distant from A and B.

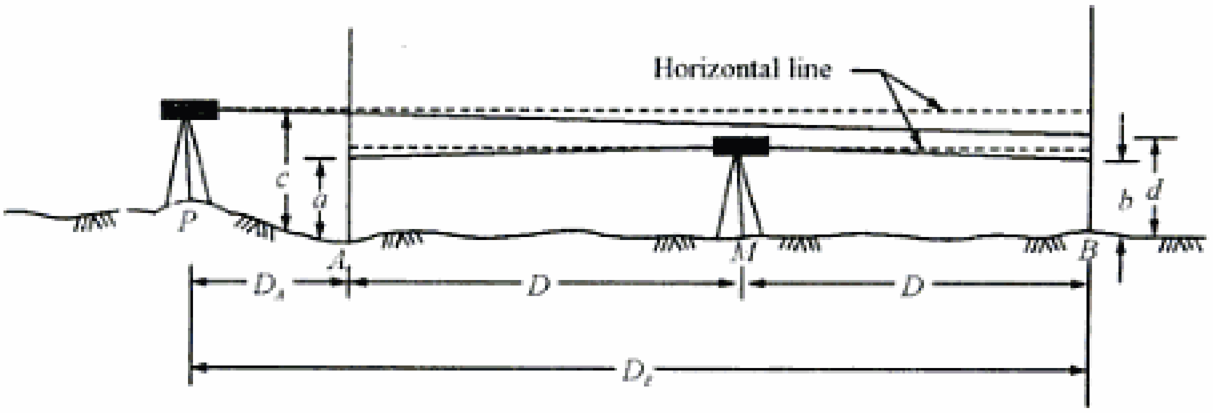


Fig. 4.23 Adjustment of the telescope level

3. Clamp the vertical circle and bring the bubble central by means vertical slow motion screw. Let the readings on the levelling staves at A and B, be a and b, respectively. Then the true difference in elevation Δh_1 will be $(a - b)$.
4. Shift the instrument to P, on the prolonged line BA, about 25 m from A.
5. With the bubble central, read the levelling staff first on A and then on B. Let the readings on A and B be c and d, respectively. Now the difference in elevation Δh_2 will be $(c - d)$.
6. If Δh_2 is equal to Δh_1, the instrument is in adjustment.
   If Δh_2 is not equal to Δh_1, the instrument is adjusted in the following manner:
   (*i*) Calculate the inclination (error) of the line of sight in the net distance $(D_B - D_A)$ which is equal to $(a - b) - (c - d)$.
   (*ii*) Calculate the error e_B in the reading on the far staff on B as under

$$e_B = \frac{D_B}{D_B - D_A}[(a-b)-(c-d)]$$

   (*iii*) Subtract algebraically the amount of the error e_B from the reading d to obtain the correct reading d' at B for a horizontal line of sight with the position of the instrument at P.
   (*iv*) Set the target at d' and bring the line of sight on the target by moving the cross hair ring vertically.

To make the vertical circle read zero when the telescope bubble is central (only for those theodolites which have a fixed vertical vernier)

This adjustment also known as adjustment of the vertical index, may be tested by bringing the plate bubble and then the telescope bubble at their centre of run, and then reading the vertical vernier. If the reading is not zero, loosen the vernier and move it until it reads zero.

Types of Theodolite

The theodolites can be classified into the following categories:

1. Transit or engineer's transit
2. Repeating theodolites, optical transit or optical reading theodolite
3. Direction theodolite.
4. Electronic theodolite.

Transit or engineer's transit

Transit or engineer's transit, most commonly referred to as a theodolite, is an instrument of great versatility, and is sometimes called the universal surveying instrument. The instruments under this category have traditionally three or four levelling screws, metal graduated scales, verniers for direct reading of horizontal and vertical angles. Because of wide variety of its uses, it has been discussed in detail in Sec. 4.6.

Repeating theodolite or optical transit

The mechanical motions of optical transit are essentially the same as those of the engineer's transit. They are equipped with graduated scales etched on glass which are viewed by way of optical trains that contain optical micrometers for measuring horizontal and vertical angles. These instruments have three levelling screws and optical plummets which replace the plumb bob.

Direction theodolite

The direction theodolite does not provide for a lower motion as contained in a repeating instrument. When it is set over a station, the horizontal circle is essentially fixed in position. Both the back sight reading and the fore sight reading are taken by means of one motion, which is the movement of the reading device with respect to the graduated horizontal circle. The difference between the back sight and the fore sight readings is directly the value of the angle.

Electronic theodolite

Electronic theodolites contain circular encoders which sense the rotations of the spindles and the telescope, convert these rotations into horizontal and vertical (or zenith) angles electronically, and display the values of the angles on liquid crystal displays (LCD) or light emmitting diode displays (LED). Further details about these instruments are given in Chapter 11 of *Higher Surveying*.

4.5 ANGLE AND DIRECTION MEASUREMENTS WITH A THEODOLITE

A single set of measurement of an angle between two points P and Q at a station O is made in the following steps (Fig. 4.24):

1. Fix two ranging poles or signals at P and Q.
2. Set up the instrument at O and perform the temporary adjustments.
3. Set the instrument for taking face left reading first by keeping the vertical circle to the left.
4. Release both the lower and upper clamp screws and turn the upper plate to set the vernier A to 0° or 360° mark on the main scale. Tighten the upper clamp screw. Bring the index of vernier A exactly to the zero of the main scale using slow motion screw of the upper plate.
5. At this stage the reading of the vernier B will be 180°. If the difference in the two readings of verniers A and B is not equal to 180°, the instrument has error due to eccentricity of verniers, or due to eccentricity of centres of spindles or both. The mean of the two readings is taken to eliminate this error. For recording the readings, the degrees, minutes and seconds on the vernier A, and only minutes and seconds on vernier B are taken.

6. Swing the telescope in the horizontal plane, and point it at the left side station *P*. Tighten the lower clamp screw, and bisect the ranging pole or signal at *P* exactly using the lower slow motion screw. The vertical slow motion screw is also used to bring the vertical cross hair exactly on the target. To minimise the error in bisection due to non-verticality of the ranging pole, the vertical cross hair should be brought to the lowest visible point of the ranging pole.
7. Loosen the upper clamp screw, and turn the telescope clockwise until the line of sight is on the ranging pole at *Q*. Tighten the upper clamp screw, and bisect the ranging pole at *Q* exactly using the upper slow motion screw.
8. Read both the verniers *A* and *B* and record the reading in the Form-1 (Table 4.1), which is a page of the field book for recording the horizontal angles. The reading of the vernier *A* is the angle *POQ*. The vernier *B* gives the value of the angle *POQ* after deducting 180°. The mean of two values of the angles obtained from the verniers A and B, is the required angle *POQ*.
9. Change the face of the instrument to the face right by transiting the telescope, and swinging it by 180°.
10. Repeat steps 4 to 8, and determine another value of the angle *POQ*.
11. The mean of the face left and face right observations, is the final required angle *POQ*.

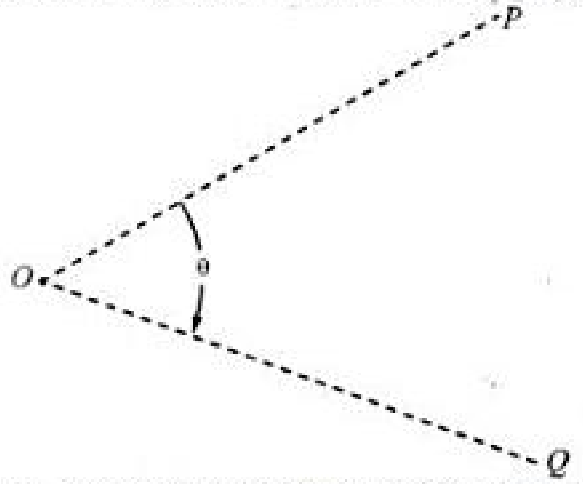


Fig. 4.24 Measurement of horizontal angle

Table 4.1 Measurement of horizontal angle

Form-1

Inst. at	Sighted to	Face Left Swing Right												Face Right Swing Left												Average horizontal angle		
		A			B		Mean			No. of repetition	Horizontal angle			A			B		Mean			No. of repetition	Horizontal angle					
		°	′	″	′	″	°	′	″		°	′	″	°	′	″	′	″	°	′	″		°	′	″	°	′	″

Common Mistakes

The mistakes often made in measuring horizontal angles are:

1. Turning the wrong slow motion screw.
2. Failing to tighten the clamp.

3. Reading the numbers on the horizontal scale in wrong direction.
4. Reading angles in the wrong direction.
5. Dropping 20′ or 30′ by failure to take the full-scale reading before reading the vernier (*e.g.*, with a circle graduated to 20′, calling an angle 35° 16′, the vernier reading being 16′ when the actual reading is 35°36′).
6. Reading the vernier in the wrong direction.
7. Reading the wrong vernier.

Horizontal Angle Measurement by Repetition Method

The method of repetition is used to measure a horizontal angle to a finer degree of accuracy than that obtainable with the least count of the vernier. By this method the value of the angle obtained is a closer value to the true value of the angle. The following example clearly explains the principle of repetition method.

Let the actual value of the angle *POQ* (Fig. 4.24) to be measured with a theodolite of 30″ least count, be 21°05′38″. For the first measurement as described in Sec. 4.5, the reading will be 21°5′30″. For the next measurement, loosen the lower clamp and bisect the ranging pole at *P* using the lower clamp and slow motion only. Loosen the upper plate and bisect the ranging pole at *Q*. The reading now will be 42°11′00″ whereas the exact reading should be 42°11′16″. The process after the third setting on the right ranging pole at *Q* should give the theodolite reading as 63°17′00″ whereas the exact reading should have been 63°16′54″. Dividing 63°17′00″ by 3, the number of repetitions, gives 21°05′40″ which is only 2″ in error.

In practice 6 to 10 repetitions with both faces readings are considered to be enough for measuring an angle to the highest degree of precision. When face left is kept, the instrument is rotated clockwise and for face right, the instrument is rotated anticlockwise. Mean of the face left and face right readings after dividing by the number of repetitions, is the final measured value of the angle.

Form-1 given in Table 4.1, may be used for recording the observations and deducing the angle by repetition method.

The following errors are eliminated by the method of repetition:

1. Taking reading on both the verniers eliminates the errors due to eccentricity of verniers and centres.
2. Taking reading at different parts of the circle, eliminates the errors due to inaccurate graduations.
3. Taking both faces readings, eliminates the errors due to inadjustment of line of collimation and the horizontal axis of the instrument.
4. Errors due to inaccurate bisection of the object, eccentric centering, etc., may be to some extent counter-balanced in different observations.

The errors due to the following are not eliminated as all these errors are of cumulative nature:

1. Slip
2. Displacement of station signal
3. Verticality of the vertical axis.

Horizontal Angle Measurement by Reiteration Method or Direction Method

When several angles are to be measured at one station, the reiteration method is used. The angle between the points *A, B, C,* and *D* shown in Fig. 4.25, are required to be measured at station *O*. Out of several stations *A, B, C, D,* etc., one may be chosen as an initial station from which the successive angles are measured. The angle between the last station and the initial station, is also measured, and this is known as *closing the horizon*. This provides a check that sum of all the angles measured at a station is 360°.

When the horizon is closed, the final reading of the vernier should be same as its initial reading. If there is any discrepancy within the permissible limits, it is distributed equally among all the angles measured. For example, if $\theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 = 360° + \delta\theta$, then the correction to each angle would be $\left(-\frac{1}{5}\delta\theta\right)$ for closing the horizon. In the case of a large discrepancy, the whole work is to be repeated.

For recording the measurements by reiteration method, Form-1 given in Table 4.1, may be used.

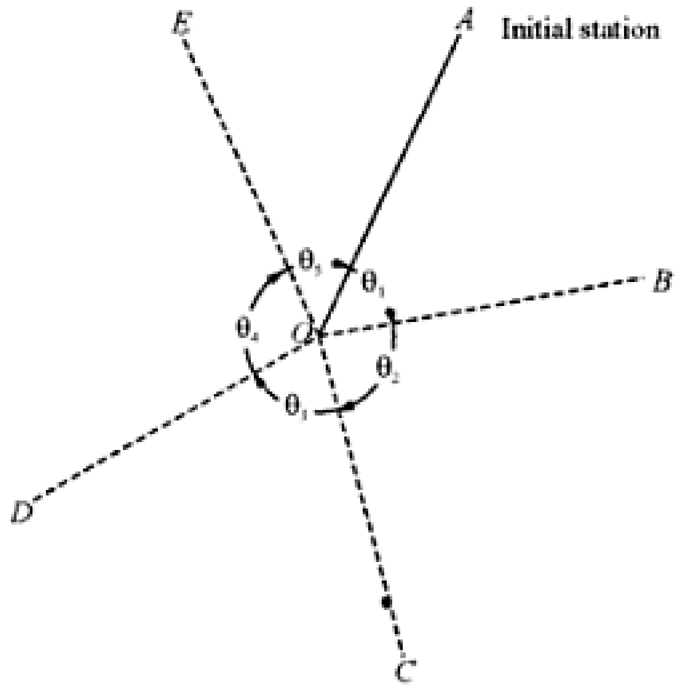


Fig. 4.25 Horizontal angle by reiteration method

When a direction theodolite is used, the instrument is set up over the selected station, and with telescope normal, the line of sight is directed at each adjacent station in turn. The telescope is rotated in a clockwise direction, and the verniers *A* and *B* are read. The telescope is then reversed, and another round of directions is observed. This entire set of readings constitutes one *position*. Depending on the precision required, number of positions may be decided. For example, for a first-order work a total of 8 to 32 positions and for a second-order work, 4 to 16 positions are observed.

The circle reading should be advanced by $180°/n$ for each new position, n being the number of positions to be observed. This is done to distribute the readings over the entire circle to eliminate the errors of graduation of the circle.

Measurement of a Vertical Angle

The method for measuring vertical angles varies with the type of instrument used. The engineer's transit measures the vertical angle with reference to horizontal plane. Direction and repeating or optical transits measure the zenith angles. The electronic theodolites may have options for measuring the vertical angles from the horizontal or from the zenith.

Using the engineer's transit, the vertical angle to a point is its angle of elevation (+) or depression (–) from the horizontal as shown in Fig. 4.26a and b, respectively.

The procedure for measuring a vertical angle is as follows:

1. After setting up, centering, and levelling up the instrument, the object to which the vertical angle is required, is bisected using the telescope clamp and slow motion screw.
2. The reading device is now set to bring the altitude bubble to the centre of its run by means of the clip screw. If the theodolite is fitted with an optical split-bubble, it is set to have an apparently continuous edge.
3. The reading on the vertical circle is now read. The reading will be the vertical angle α, $(180° - \alpha)$, $(90° + \alpha)$, or a similar function of α, depending on the graduation system used for the vertical circle.

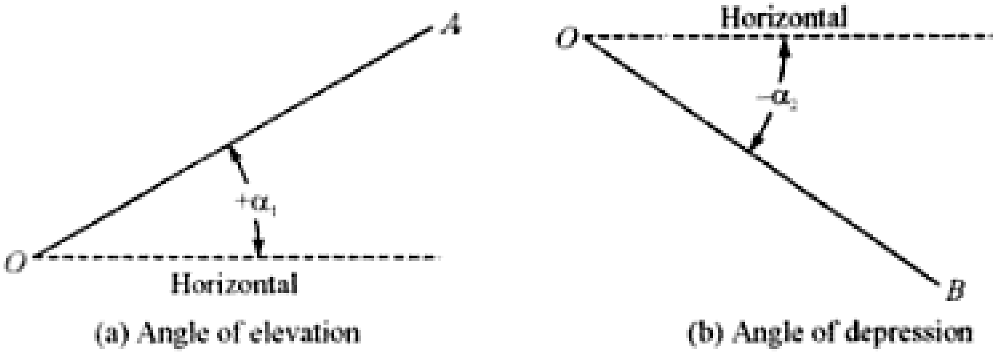


Fig. 4.26 Vertical angle

When several vertical angles are to be measured at one station, the altitude bubble is set to its centre of run for all pointings of the telescope in step 2 above. To achieve this, the procedure given below is followed:

1. Turn the instrument about the vertical axis so that the altitude level is parallel to the line joining any two levelling screws. Bring the bubble to its centre of run by turning the levelling screws either both inwards or both outwards.
2. Turn the telescope through 90° so that the altitude level is perpendicular to the line joining the same levelling screws. Bring the bubble of the altitude level again to the centre of its run by turning the third screw.

3. Turn back the telescope through 90° so that the altitude level is again parallel to the two levelling screws.
4. Repeat steps 2 and 3 until the bubble of the altitude level remains central in both the positions.
5. Now swing the telescope through 180° so that the altitude level is parallel to the two levelling screws but the eyepiece-end and the objective-end are reversed.The bubble will remain central if the instrument is in permanent adjustments, and it will remain central for all pointings of the telescope.

The vertical angle observations may be recorded in Form -2 given in Table 4.2.

Table 4.2 Measurement of vertical angle

Form-2

		Face Left													Face Right														
Inst. at	Sighted to	±	*C*			*D*		Mean			No. of repetition	Vertical angle			*C*			*D*		Mean			No. of repetition	Vertical angle			Average vertical angle		
			°	′	″	′	″	°	′	″		°	′	″	°	′	″	′	″	°	′	″		°	′	″	°	′	″

While measuring vertical angles, the following precautions must be taken:

1. For computing the reduced levels of the objects, it is essential to know whether the measured vertical angle is the angle of elevation or depression. Therefore, care should be taken to note +ve sign for angles of elevation and –ve sign for angles of depression.
2. Both the verniers must be read to obtain the mean vertical angle.
3. To eliminate or minimise the errors due to inadjustments of the instrument, both face readings should be taken.

4.6 FIELD OPERATIONS WITH A THEODOLITE

Theodolite is a versatile instrument, and several field operations other than the measurement of horizontal and vertical angles, are also carried out with it. Some of field operations which can be performed with theodolite, are discussed below. In fact, many other tasks can be performed by a theodolite, depending on the ingenuity of the surveyor. In all the measurements discussed below, it is a general practice to read both the verniers, repeat the work on both faces of the instrument, and take mean of all these values.

Measurement of Direct Angles

Direct angles are the angles to the right or azimuths from the preceding lines (*cf.*, Sec. 4.2). They are commonly used in open traverses (*cf.*, Sec. 4.7). These angles range between 0° to 360°.

The general procedure to measure the direct angles is to back sight on the preceding station with zero reading on one of the verniers, then turn it clockwise, sight the forward station and take the reading on the horizontal circles. This reading is the direct angle at the station.

Measurement of Deflection Angles

The measurement of deflection angles (*cf.*, Sec. 4.2) is done by taking a back sight on the previous station with zero reading on one of the verniers. Then the telescope is transited and turned clockwise or anticlockwise as the case may be, the forward station is sighted and the horizontal circle is read. If the deflection angle is to the right, the reading itself is the measured value of the angle, otherwise the difference between 360° and the reading, gives the deflection angle to the left.

Measurement of Magnetic Bearing

Magnetic bearings (*cf.*, Sec.4.2) may be measured with the aid of some form of compass fitted to the theodolite. The whole-circle bearing is measured by right swing, *i.e.*, turning the telescope clockwise. The following is the procedure to measure the whole-circle bearing of a line.

Set up the instrument, complete the temporary adjustments at the station and set one of verniers to zero. Loosen the lower clamp and the magnetic needle is released. Turn the telescope until the magnetic needle takes its normal position indicating the N–S direction. In this position of the instrument, the telescope points to the magnetic north while the reading is zero. Clamp the lower plate to loosen the upper clamp, and turn the telescope clockwise to sight the forward station. The reading on the horizontal circle now is the magnetic whole-circle bearing of the line.

Lining-in

Lining-in is the process of establishing intermediate points on a given line (Fig. 4.27) and is similar to ranging discussed in Sec. 3.4.1. It is assumed that the end points *A* and *B* are intervisible.

Set up the instrument over *P*, centre and level it accurately. Clamp the upper plate, and turn the telescope until *Q* is bisected. Clamp the lower plate, and bisect *Q* exactly using the slow motion screw. Ask the assistant to fix the station marks at 1, 2 and 3, such that they coincide exactly with the intersection of the cross hairs. The work should be repeated on the other face of the instrument to check the discrepancy which should then be set right by choosing the mean position for each of the intermediate points.

Fig. 4.27 Lining-in

Balancing-in

Method of establishing intermediate points on a line whose extremities are not intervisible but they are visible from some intermediate point, is known as *balancing-in*. It is also called *wiggling-in*.

In Fig. 4.28, the points A and B are the extremities of a line AB which are not intervisible due to intervening ground, but they are visible from the intermediate point M.

Set up the instrument at an intermediate point M_1 as close to the line AB as can be estimated. Take a backsight on A. Plunge the telescope and locate the point B_1 where the line of sight strikes near B. Measure the distance B_1B, and determine M_1M by which the instrument has to be shifted laterally. So that the point M is on the line AB.

$$M_1M = \frac{B_1B}{L}\, l \tag{4.1}$$

The distance l is measured and the distance L is found out from the known coordinates of A and B, or from the map.

Now set up the instrument at the estimated position M_2 which may not be at the required location M. Take a backsight again on A, plunge the telescope, and locate the point B_2. Shift the instrument laterally by a distance M_2M which is now computed from

$$M_2M = \frac{B_2B}{L}\, l \tag{4.2}$$

Repeat the above process until the line of sight strikes B when the telescope is plunged. The instrument station is then the required intermediate point M on the line AB. Locate the point M vertically beneath the centre of the instrument using the plumb bob or optical plumet.

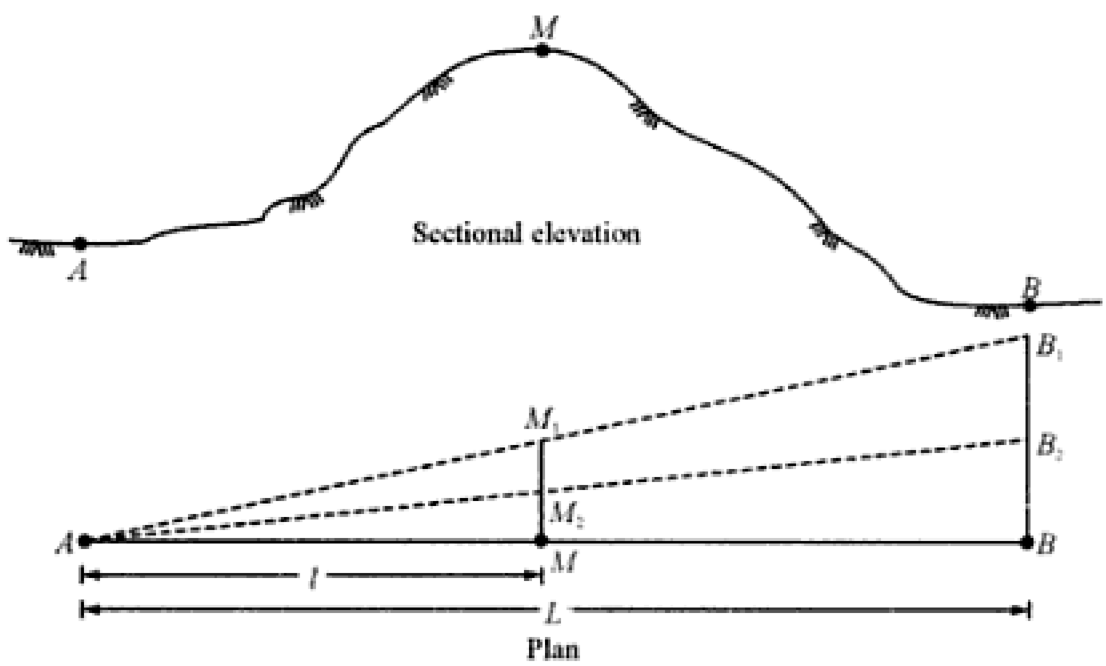


Fig. 4.28 Balancing-in

Double Sighting: If the intermediate point is to be established with high precision or when the instrument is not in perfect adjustment, the point M is obtained by the method of double sighting. In this method, two points M' and M'' are located by both face observations, *i.e.*, telescope normal and telescope inverted. The mean of the two positions is the required point M (Fig. 4.29).

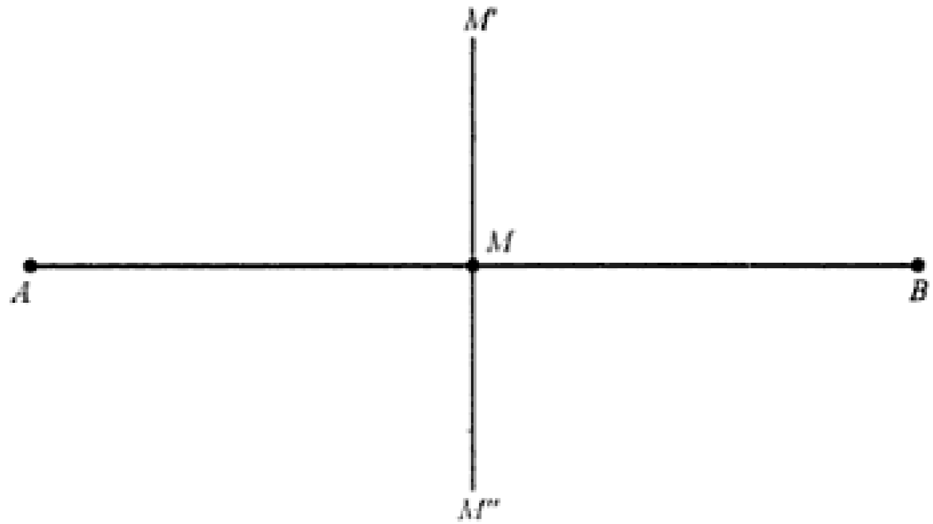


Fig. 4.29 Double sighting

Random Line Method of Running a Line

If the ends A and B of a line are neither intervisible nor are they visible from an intermediate points, the random line method is used for running the line AB. These conditions arise when the area is thickly wooded or hilly. The line AC shown in Fig. 4.30, is called the *random line* or *a trial line*.

Locate a point C near to B from where both A and B are visible. Set up the instrument at C, and measure the angle ACB. Let this angle be θ. Measure the distances AC, and CB with a tape. Now locate a point p on the line AC, and set up the instrument over the point p. Lay off the angle ApP equal to θ. Measure the distance pP equal to $BC \dfrac{Ap}{AC}$. The point P is on the line AB. Other points Q, R, etc., are also located in the similar manner.

Check whether the line of sight strikes P, Q, and R exactly when the instrument is at A or B. If any point is found not to lie exactly on the line AB, it is shifted laterally to bring it on the line.

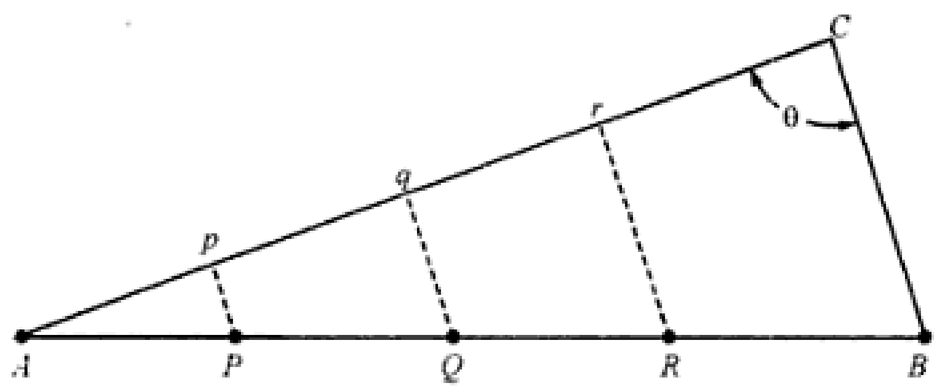


Fig. 4.30 Random line method of running a line

Prolonging a Straight Line

It is often necessary to prolong or extend a line for a significant distance by locating an adequate number of points along the line. There are three methods for prolonging a line and they are illustrated below.

Method-I (Fig. 4.31)

Set up the instrument over the end A, and sight the end B of the line AB after the horizontal motion of the instrument has been fixed. Locate a point C at a reasonable distance. Shift the theodolite to B and locate another point D on the prolongation of BC. The process is repeated until the line is prolonged to the disired distance. This method will result in cumulative errors if the instrument is not in adjustment.

Fig. 4.31 Prolonging a line by first method

Method-II (Fig. 4.31)

Set up the instrument over B and backsight A. Transit the telescope and locate C. Shift the instrument to C and backsight B. Transit the telescope and locate D.

The process is continued until the line is prolonged to the desired distance. This method is superior to the first because the distance involve each time in the prolongation process is reduced to a uniform value of the order of AB, and the error is not carried over to other spans.

Method-III (Double Sighting) (Fig. 4.32)

Set up the instrument at B, backsight A with face left, plunge the telescope, and locate C_1. Change the face of the instrument to right, backsight A again, plunge the telescope and locate C_2. The point C_2 will be obtained only when the instrument is out of adjustments. The mean of the two locations C_1 and C_2 is the desired point C. The process is repeated and continued until the line is prolonged to the distance required. In this method, the error is doubled on reversal of the telescope, and the mean of two locations becomes the desired location of the point on the prolongation of the line. This method is used when high precision is required, and the instrument is suspect with regard to adjustment.

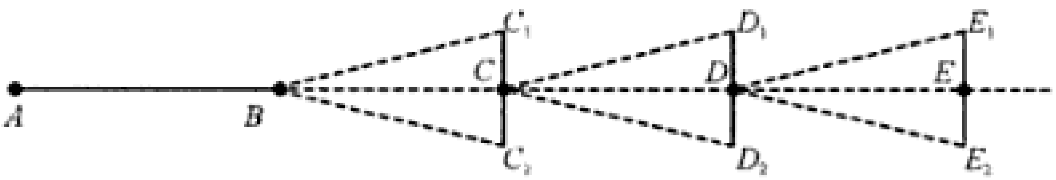
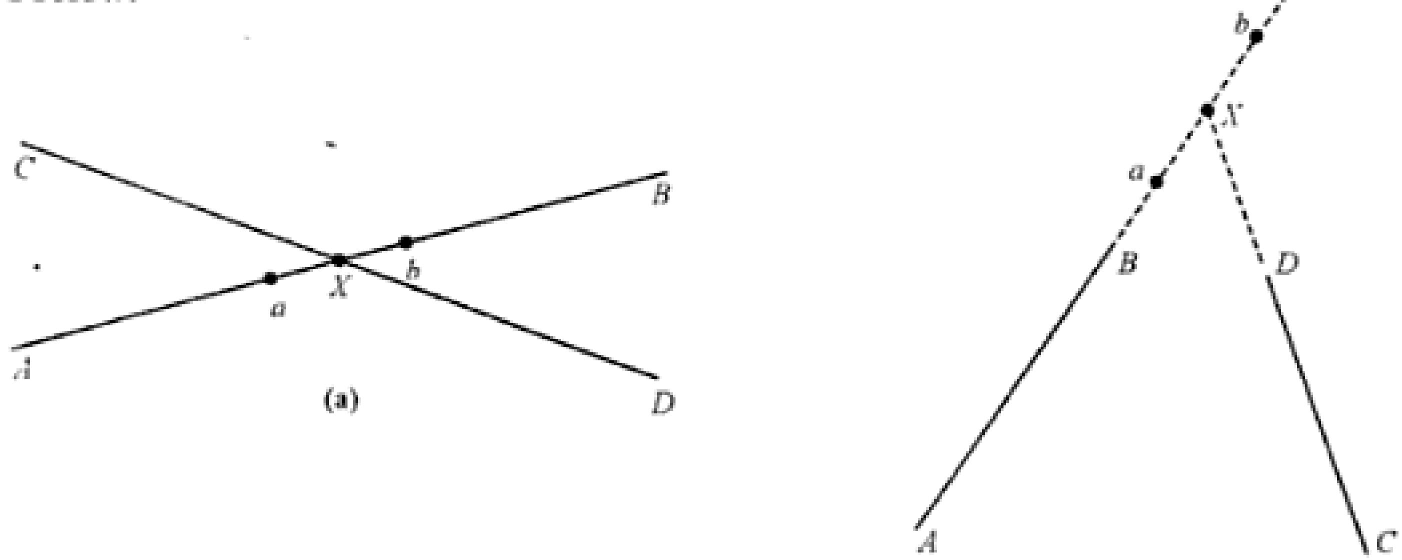


Fig. 4.32 Prolonging a line by double sighting

Location of Intersection of Two Straight Lines

There may be two cases of intersection of lines. In the first case, the intersection may fall on the junction of the lines (Fig. 4.33a), and in the second case, it may be on the prolongation of line (Fig. 4.33b).

The intersection X of the two straight lines AB and CD can be located using a theodolite, as explained below:



(b)

Fig. 4.33 The intersection of two lines

Case-I (Fig. 4.33a): Set up the instrument at one end of one of the lines, say at *A* on *AB*, and sight *B*. Clamp the horizontal motion and fix two points *a* and *b*, a small distance apart on either side of the estimated position of the intersection *X* with wire nails or tacks. Stretch a string between the points *a* and *b*. Now shift the instrument to the end *C* on the other line *CD*, and sight *D*. The point of intersection *X* is marked on the string where the line of sight cuts the string.

Case-II (Fig. 4.33b): Set up the instrument at *A*, and sight *B*. Fix the points *a* and *b* on the prolongation of *AB* on either side of the estimated position of the intersection *X*, and stretch a string between them. Now shift the instrument to *C*, the farther end of *CD*, and sight *D*. The prolongation of *CD* cuts the string at the intersection *X* of the two lines.

To achieve a better accuracy in locating the intersection, double sighting can be used in both the above cases.

4.7 THEODOLITE TRAVERSING

A traverse consists of a series of straight lines connecting successive established points along the route of a survey. The points defining the ends of the traverse lines are called *traverse stations*, and the processes of locating the traverse station is *traversing*. Traverse is required to be run in land surveys, city surveys, hydrographic surveys, and locations surveys in highways, railways, canals, etc.

When a traverse is run by using a theodolite, it is called theodolite traversing. Traverse can also be run by using a compass, a tacheometer or a plane table, and the process is known as compass traversing, stadia traversing, and plane-table traversing.

The traversing has been discussed in detail in Chapter 9.

4.8 TO LAY OFF A HORIZONTAL ANGLE

The laying off a horizontal angle is required sometimes in construction works, and the process is the reverse of process of that of measuring the angles.

To lay off an angle *AOB* equal to θ, as shown in Fig. 4.34, set up the instrument at *O*, and set the vernier *A* to read 0° or 360° using upper clamp and slow motion screw. Turn the telescope, and sight the station *A*. Clamp the lower plate, and bisect *A* accurately using the slow motion screw. Now loosen the upper clamp, and turn the upper plate until the vernier *A* reads the given angle θ. Use of slow motion screw is made to set the given angle accurately.

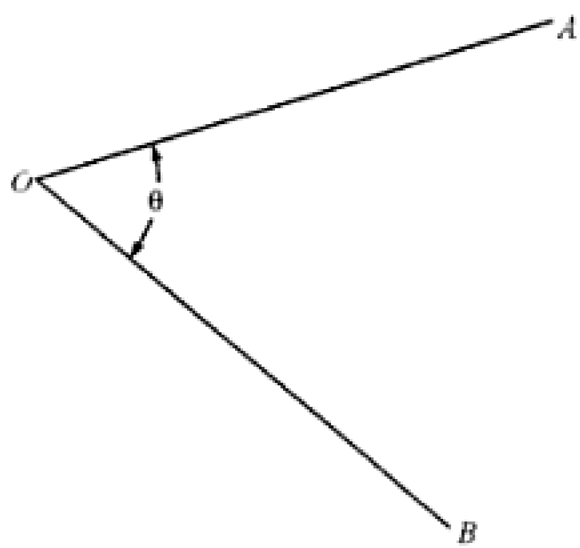


Fig. 4.34 Horizontal angle

The line of sight is now in the required direction *OB*. Depress the telescope and establish *B* on the ground along the line of sight. The angle *AOB* laid is equal to the angle θ.

When a horizontal angle is to be laid off with higher precision, the method of repetition should be used in similar manner as discussed above.

4.9 ERRORS IN THEODOLITE MEASUREMENTS

The sources which cause errors in the measurements made with the theodolites, may be classified as below:

1. Natural errors
2. Personal errors
3. Instrumental errors
4. Mistakes.

Natural Errors

Sources of natural errors are: (1) temperature, (2) wind, (3) refraction, and (4) settlement of tripod. In general, the natural errors are not very large enough to affect appreciably the measurements of ordinary precision. These errors are generally random, but under certain conditions they may be systematic in nature. For surveys of very high precision, special attempt is made to establish a procedure which will as nearly as possible minimise or eliminate systematic errors.

Temperature effect

Effect of temperature difference may be unequal expansion of various components of a theodolite leading to errors, It can be minimised by shielding the instrument from heat.

Wind effect

Effect of strong winds is to create vibrations in the instrument which makes it difficult to work. Also the plumb bob gets deflected making the accurate centering difficult. To reduce the wind effect, the instrument is shielded from the strong winds. Alternatively, the work can be suspended during the strong winds.

Refraction effect

Shimmering of the signals due to refraction is experienced when the line of sight is close to the ground. It makes the accurate bisection difficult. To reduce the refraction effect, the line of sight is always kept well above the ground surface. Also sights close to buildings, smoke stacks, and even large individual bushes are to be avoided.

Settlement of tripod

Due to the weight of the instrument, the settlement of the tripod may take place if the ground is soft. In such cases, stakes may be driven to support the tripod legs or a triangular frame may be used to support the tripod.

Personal Errors

Personal errors arise from the limitations of the human eye in setting up and levelling the instrument and in making the observations. These errors are random and hence cannot be eliminated.

Errors due to inaccurate centering

If the instrument is not set up exactly over the station, it produces an error in all angles measured at the given station. The magnitude of the error varies directly with the direction of pointing and inversely with the length of sight. The error may be kept within negligible limits by taking reasonable care. When the sights are long, unnecessary time should not be wasted in setting up. Centering to within 6 to 9 mm is possible without difficulty using a plumb bob. When the instrument is provided with an optical plummet, centering errors less than 1 mm are possible.

Errors due to inaccurate levelling

This error is small when the sights are nearly level, but may be large for steeply inclined sights. This error may be minimised or eliminated by frequent checking of the position of the bubble centre, and if necessary it should be recentered. Relevelling should never be done between a back sight and fore sight.

Slip in screws

An error will occur if the clamp screws are not properly tightened. The magnitude of error will depend upon the slip. The error is avoided by properly tightening all the screws.

The slip may also occur when the shifting head is not properly tightened or when the instrument is not properly fixed to the tripod head.

Improper use of tangent screws

The error is introduced by improper use of the screws. While taking the back sights, the lower slow motion screw should be used and for the fore sights, upper slow motion screw. The slow motion screws should only be used after tightening the corresponding clamp screws. To avoid error due to backlash, the final bisection of the object should be achieved only by turning the slow motion screws in positive direction.

Errors in setting and reading the vernier

These functions are of the least count of the vernier, and of the legibility of scale and vernier lines. A magnifying glass can be used to enable closer reading. The vernier error may be reduced by using the method of repetition.

Inaccurate sighting

Inaccurate sighting is likely to be a source of rather large error in ordinary surveys when the sights are taken on the ranging pole of which often only the upper position is visible from the instrument.

The error can be minimised by accurately centering the vertical cross hair on the signal, and sighting the lowest portion of the ranging pole placed at the station mark. For short sights greater care is required than for long sights. If the station to be observed is quite close, the string of the plumb bob should be used in place of ranging pole for accurate sighting.

Parallax

The error due to imperfect focussing is always present to greater or lesser degree but can be reduced to a negligible quantity by taking reasonable care in focussing.

Instrumental errors

Instrumental errors are those errors which are due to instrumental imperfections and / or inadjustments. They are systematic errors, and can be either eliminated or reduced to a negligible amount.

Error due to imperfect adjustment of the plate level

This error causes the measurement of horizontal angles in an inclined plane instead of a horizontal plane, and cannot be eliminated by averaging the face left and face right readings. The permanent adjustment of the instrument has to be done to eliminate this error.

Error due to line of sight not being perpendicular to horizontal axis

If the line of sight is not exactly perpendicular to the horizontal axis of the instrument, the telescope does not revolve in vertical plane, instead it generates a cone.

The error in horizontal angle occurs if the angle of inclination of the back sight is not equal to that of the fore sight. The error is eliminated by averaging the face left and face right readings, *i.e.*, by double sighting.

Error due to horizontal axis not being perpendicular to the vertical axis

This error is similar to the error due to line of sight not being perpendicular to horizontal axis discussed above, and it is also eliminated by averaging the face left and face right readings.

Error due to eccentricity of inner and outer axes

This error is eliminated by averaging the readings taken on both the verniers.

Error due to eccentricity of verniers

The error due to zero marks of the verniers *A* and *B*, not being at the ends of the same diameter and not being exactly 180° apart, is also eliminated by taking mean of readings on verniers *A* and *B*.

It may be noted that eccentricity of verniers does not introduce any error in horizontal angles so long the same vernier is used for the initial and final readings.

Errors due to imperfect graduations

Errors of this source can be minimised to a negligible amount by taking the mean of several observations for which the readings are distributed over the circle and over the vernier.

Error due to non–adjustment of vertical vernier or vertical index error

Non-adjustment of vertical vernier introduces a constant error in the vertical angles. The error is eliminated by averaging the face left and face right readings.

Mistakes

The common mistakes or blunders which generally the surveyors make due to carelessness, are:

1. Misreading the vernier.
2. Reading the wrong vernier scale.
3. Turning wrong tangent screws.
4. Sighting on the wrong signal, or setting up the instrument over the wrong station.
5. Booking wrong values of the readings.
6. Missing sign (+ or –) for vertical angles.
7. Missing the word right or left for deflection angles.

The mistakes can only be avoided by adopting suitable field procedures.

4.10 ANGLE MEASUREMENT USING A TAPE

It is possible to make simple surveys by using just a tape. If the sides of a triangle are measured, sufficient data are obtained for computing the angles of a triangle. The error in the computed values of the angles depends on the care with which points are established and the precision with which measurements are taken. The method is slow and is generally employed as a check.

4.11 ANGLE MEASUREMENT USING PLANE TABLE AND ALIDADE

A plane table is a drawing board mounted on a tripod, and an alidade is a straight edge with some form of sighting device. Using a plane table and an alidade, angles can be determined graphically. The instrument and the method have been discussed in Chapter 10.

4.12 ANGLE MEASUREMENT USING A SEXTANT

Sextant is an instrument generally used for hydrographic surveys. An observer can measure the angles while he is moving, hence, angles can be read from a boat. The angles measured by a sextant are not

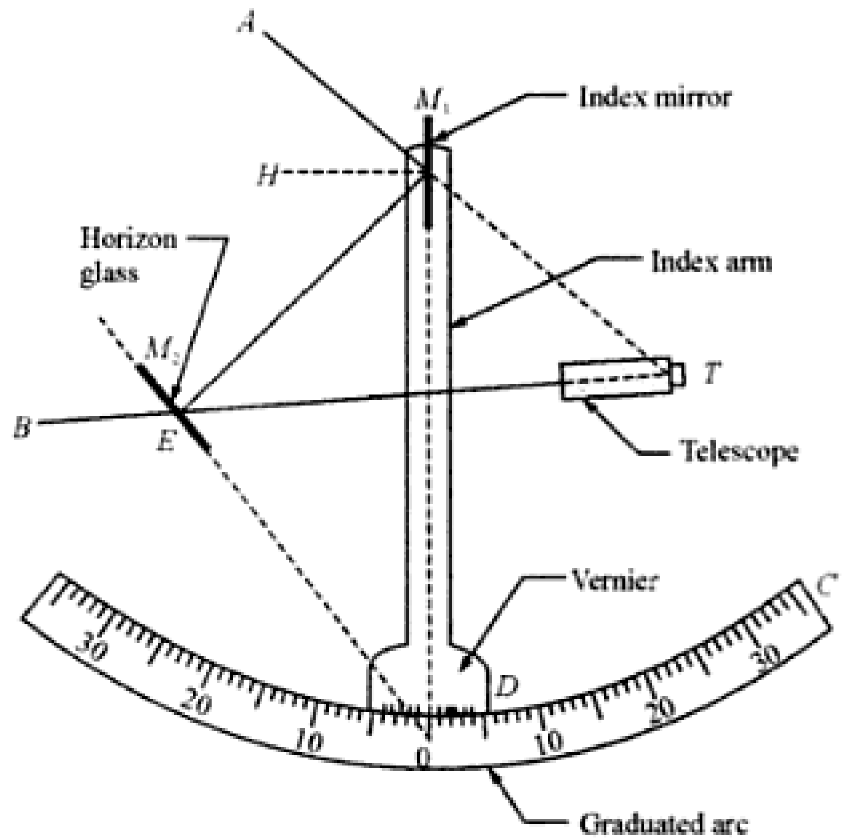


Fig. 4.35 Essential features of a sextant

generally in the horizontal plane and, therefore, are not horizontal angles. Sextant is not an instrument of high accuracy for angles less than 15° and distances less than 300 m.

The essential features of a sextant are shown in Fig. 4.35. An index mirror M_1 is rigidly attached to a movable arm M_1D, which is fitted with a vernier, clamp, and a slow motion screw or tangent screw. The movable arm moves along the graduated arc shown in the figure. A second mirror, M_2, known as horizon glass, the lower portion of which is half silvered and the upper half clear. The mirror M_2 and a telescope T are rigidly attached to the frame. The telescope points towards M_2.

Let the angle to be measured is ATB. T is also the observer's position. The ray of light from signal at B passes through the clear portion of M_2 on through the telescope to the eye at T. The ray of light from signal at A strikes the index mirror at M_1, and is reflected to E and then through the telescope to T. Each set of rays forms its own image on its respective half of the objective. By moving the M_1D these images can be made to move over each other. There will be one position in which they coincide, and the vernier reading on the graduated arc C is the required angle ATB.

Measuring Angles with the Sextant

To measure the angle ATB (Fig. 4.35), hold the sextant in the right hand and adjust the index arm to read zero. Sight the left-hand object B through the telescope, and the clear portion of the horizon glass M_2. Move the index arm with the left hand until the image of second object A coincides with the image of B. The final setting is made with the tangent screw. The test for the exact coincidence is made by twisting the sextant slightly to make the reflected image move back and forth across the position of coincidence. Take the reading on the graduated arc using the vernier, which is the value of the desired angle between A and B at the instrument station.

4.13 ANGLE MEASUREMENT USING A MAGNETIC COMPASS

The magnetic compass is useful for rough surveys and retracing early land surveys. The compass mounted on a theodolite helps in approximate checking of measured horizontal angles. A detailed discussion on compass and its uses are presented in Chapter 5.

PROBLEMS

4.1 Describe the methods of locating points on or near the surface of the earth.

4.2 Describe the means of defining the angles and directions.

4.3 Define the following:

(*a*) Bearings
(*b*) Azimuths
(*c*) Deflection angles
(*d*) Angles to right
(*e*) Interior angles.

4.4 What is a meridian? Differentiate between true, grid, magnetic, and arbitrary meridians.

4.5 Describe various types of bearings.

4.6 Explain the difference between the whole-circle bearing and reduced bearing of a line.

4.7 Express the following bearings as (a) azimuths from the north and (b) azimuths from the south:

N 43°20′ E, S 36°42′ E, N 40°30′ E, S 35°52′ W, N 77°40′ W, N 89°49′ E, N 63°28′ W, S 79°59′ W, N 8°15′ W, S 12°18′ E.

4.8 Following are the forward azimuths: $A_{AB} = 185°30'$, $A_{AC} = 145°45'$, $A_{CD} = 205°40'$, $A_{DE} = 18°59'$, $A_{AF} = 98°15'$.

Convert these directions to back azimuths.

4.9 The following azimuths of the lines are reckoned from the north:

$AB = 162°19'$, $BC = 279°33'$,

$CD = 333°50'$, $DE = 5°35'$.

What are the corresponding bearings? Also determine the deflection angles between the consecutive lines.

4.10 Draw a neat sketch of a transit, and explain the functions of its various components.

4.11 Differentiate between

(*a*) Transiting or plunging and swinging of the telescope.
(*b*) Face left and face right readings.
(*c*) Clamp screw and tangent screw.
(*d*) Telescope normal and telescope inverted.

4.12 What are the advantages of an internal focussing telescope?

4.13 Define the following terms associated with the angle measurements with a theodolite:

(*a*) Vertical axis; (*b*) Trunion axis; (*c*) Axis of the plate level; (*d*) Centering; (*e*) A set.

4.14 What do you understand by 'Temporary Adjustments'? Describe in brief the various temporary adjustments of a theodolite.

4.15 Discuss the relationships between the various axes of a transit.

4.16 What are the permanent adjustments in a theodolite? Discuss briefly their effects on the angle measurements.

4.17 Describe the various types of theodolite.

4.18 Discuss the procedure of measuring a horizontal angle by repetition. What are its advantages?

4.19 How would you measure horizontal angles by reiteration method?

4.20 Discuss the procedure of measuring a vertical angle using a transit.

4.21 Write briefly the procedures for the measurement of (*i*) Direct angles; (*ii*) Magnetic bearings; (*iii*) Deflection angles.

4.22 Write short notes on the following:

(*i*) Lining-in; (*ii*) Balancing-in; (*iii*) Random line; (*iv*) Prolonging; (*v*) Location of intersection; (*vi*) Double sighting.

4.23 How would you lay off a horizontal angle using a theodolite? If an angle is required to be laid off with higher precision, what method is to be used?

4.24 What are different types of errors in theodolite surveying? How are they eliminated or minimised?

4.25 Describe the various components of a sextant. Explain the method of measuring angles with a sextant. Why the angles measured with a sextant may not be true horizontal angles?

4.26 A theodolite having graduations 0° to 360° in clockwise direction on its circle, is used to measure an angle by 12 clockwise repetitions, 6 readings taken with telescope normal and 6 readings with telescope inverted. The observations are as under:

Telescope	Reading	Vernier	
		A	B
Direct	Initial	48°46′	228°46′
Direct	After first turning	161°9′	
Reversed	After twelfth turning	317°24′	137°25′

Compute the value of the angle.

4.27 A direction theodolite with a least count of 1″, is set over a station S to measure directions to station R, Q, and P. The observed directions are as given below:

Station	Telescope	Reading
R	*D* *R*	0° 10′16″ 180°10′26″
Q	*D* *R*	48°52′06″ 228°52′40″
P	*D* *R*	83°06′48″ 263°06′48″
D = Direct	R = Reverse	

Compute the average angles RSQ and QSP.

5

CHAIN AND COMPASS SURVEYING

5.1 GENERAL

The purpose of land surveying is to secure necessary data of a parcel of land for the purpose of defining or demarcating its boundaries, determining its area, preparing its plan or map, or execution of an engineering project. The collection of data can be done by any one or combination of surveying methods, *e.g.*, chain surveying, compass surveying, plane table surveying, tacheometric surveying, theodolite surveying, photogrammetry, or remote sensing, depending on accuracy required, extent and type of the area to be surveyed, time available for the survey work, and economic aspects. Once a work is assigned to the surveyor, he has to choose the best approach so that work can be completed within the given time-frame and available funds.

5.2 CHAIN SURVEYING

Chain surveying is the simplest method of surveying in which only linear measurements are made and no angular measurements are taken. The area to be surveyed is divided into a number of triangles, and the sides of the triangles are directly measured in the field as shown in Fig. 5.1. Since a triangle is a simple plane geometrical figure, it can be plotted from the measured lengths of its sides alone. In chain surveying, a network of triangles is preferred.

Preferably, all the sides of a triangle should be nearly equal having each angle nearly 60° to ensure minimum distortion due to errors in measurement of sides and plotting. Generally, such an ideal condition is practically not possible always due to configuration of the terrain and, therefore, attempt should be made to have *well-conditioned triangles* in which no angle is smaller than 30° and no angle is greater than 120°. The arrangement of triangles to be adopted in the field, depends on the shape, topography, and the natural or artificial obstacles met with.

Chain surveying is suitable in the following cases:

1. Ground fairly level and open with simple details
2. Large-scale plans
3. Extent of the area comparatively small.

Chain surveying is unsuitable in the following cases:

1. Area crowded with many details
2. Wooded countries

3. Undulating areas
4. Extent of area large.

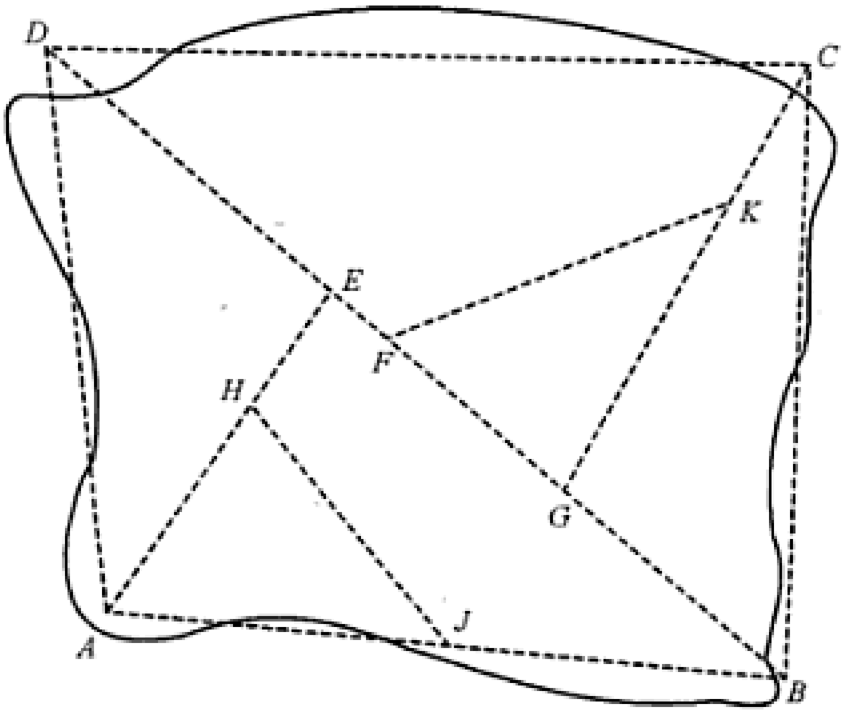


Fig. 5.1 Network of triangles in chain surveying

Definitions of Technical Terms Used in Chain Surveying

Main survey stations: A main survey station is a point where two sides of a triangle meet. These are the points at the beginning and at the end of a chain line.

Tie stations or subsidiary survey stations: These are the points selected on the main survey lines for running auxiliary lines.

Main survey lines: These are chain lines joining the main survey stations.

Tie or subsidiary lines: The chain lines joining the tie stations or subsidiary stations are tie or subsidiary lines. These lines help in locating the interior details which are far away from the main survey lines.

Base line: The longest of the main survey lines is known as the base line. Various survey stations are plotted with reference to this line.

Check lines: Those lines which are run to check the accuracy of the field work are called as check lines or proof lines. The length of a check line in the plan must agree with its measured length in the field. Each triangle must be provided with a check line.

Offsets: Offsets are lateral distances measured from the survey lines. Offsets are taken from the survey lines to determine the locations of details. These may be perpendicular offsets or oblique offsets.

The *perpendicular offset* or simply offset, is perpendicular to the survey line. It is the perpendicular distance of a detail from the survey line as shown in Fig. 5.2. The *oblique offset* or *tie* is a short measurement inclined to the survey line.

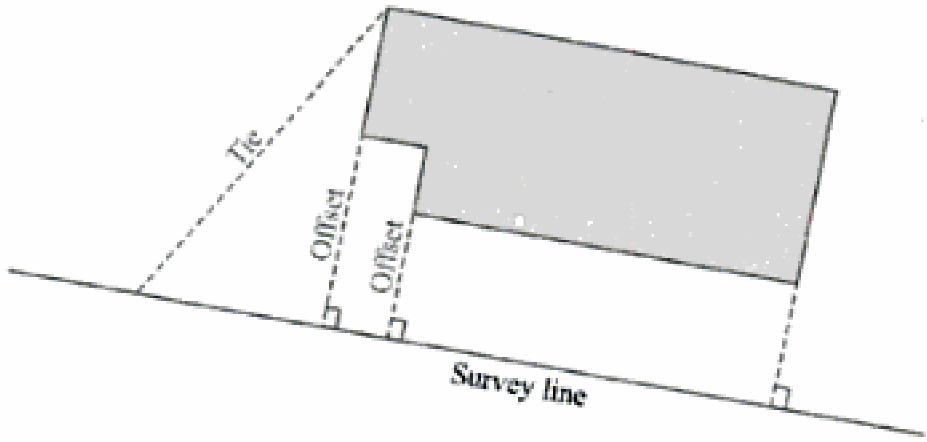


Fig. 5.2 Perpendicular and oblique offsets

In Fig. 5.1, the surveying stations, survey lines, etc., are as below:

Main survey stations	: *A, B, C, D*
Tie or subsidiary stations	: *E, F, G*
Main survey lines	: *AB, BC, CD, DA*
Tie or subsidiary lines	: *AE, CG*
Base line	: *BD*
Check lines	: *KF, HJ*

Planning and Carrying Out Chain Survey

In the field, chain surveys involve reconnaissance, selection of survey stations, and running the survey lines. These operations are described below.

Reconnaissance

Reconnaissance is the preliminary inspection of the area to be surveyed to collect preliminary information for planning and executing the survey work in the best possible manner. During reconnaissance, the surveyor prepares a rough sketch showing the arrangement of survey lines and other salient features of the network.

Selection of stations

Main survey stations at the ends of chain lines must be intervisible, and should be minimum possible in number to make the number of survey lines a minimum. Preferably these stations should be near the boundary of the area to be surveyed so that the principle of working from the whole to the part is observed. The base line should run through the centre of the area. The survey stations should form well-conditioned triangles and every triangle should be provided with a check line. To avoid long offsets, tie lines should be run and also the larger sides of the triangles should be laid parallel to boundaries, roads, buildings, etc. Tresspassing should be avoided by placing the main survey lines within the boundaries of the property to be surveyed. All survey lines should lie over level ground as far as possible, and obstacles to chaining or ranging, if any, should be avoided.

Running survey lines

The work of running a survey line comprises the chaining of the line, ranging, and the location of adjacent details from it by taking offsets. The distances along the survey line at which streams, roads, fences, and other salient features are intersected, are also noted down. The methods of measurement of horizontal distances have already been discussed in Chapter 3.

Recording Field Notes

For recording the measurements taken in the field, a field book which opens lengthwise is used. A single red line or two red lines are ruled on each page in the field book as shown in Fig. 5.3.

The chainages of various points are written in the column between the red lines. The field books containing single line, are more useful when the scale of plotting is going to be large and many details are to be entered. The offset distances are noted on the side corresponding to which they are taken. A fresh page is used to commence each survey line whose designation is noted at the bottom and booking proceeds from the bottom of the page upwards. As various features are reached, they are sketched and the offsets taken to them are noted on the appropriate side, left or right, of the survey line along with the chainage at which the offsets are taken. The surveyor should take extreme care while recording the various measurements, and should not leave anything to memory.To prevent the notes from being washed out by rains, recording should be done using a hard pencil (3H or 4H).

Plotting a Chain Survey

For plotting maps or plans of the area surveyed, the following are the steps:

1. Selection of scale
2. Plotting of framework
3. Plotting of offsets
4. Title, scale, legends, etc.
5. Inking in
6. Colouring.

Selection of scale of the map

Generally the scale of plotting is decided well before the commencement of the field work to maintain consistency in making the linear measurements and taking offsets.

Plotting of framework

The survey lines are first plotted to the scale on a tracing paper. A good quality drawing paper is taken, and the tracing paper is so oriented on the drawing paper that the framework of the triangles is centered and oriented properly. The ends of the base line are pricked through the tracing paper with a sharp pin and then the tracing paper is removed. Join the ends of the base line obtained after pricking on the drawing paper.

The intermediate stations on the base line are marked and the position of other stations is then located by drawing arcs from the ends of the base line. The triangles are checked by drawing check lines and if any error is found, it is adjusted.

Plotting of offsets

After plotting the framework of triangles the details are plotted by taking offsets from the respective survey line using set square or offset scale. The details are shown in a map or plan using the conventional symbols. Some of the common conventional symbols are given in Fig. 5.4.

Title, scale, legend, etc.

The title of the plan, north direction, representative fraction (R.F.) of the scale of the plan, a graphical scale, and a legend giving the details of the conventional symbols should be given in the plan at appropriate places.

Inking in

After plotting the complete plan in pencil, the plan is sometimes inked in black. The chain lines are shown by dash and dot in red ink and offsets and tie lines are not shown in the final drawing. The main stations are indicated by circles of about 6 mm diameter.

Colouring

If the plan is to be coloured, the drawing is cleaned with bread crumbs before commencing the colouring. The colours for various symbols/details are given in Fig. 5.4.

Obstacles in Chaining

While chaining a survey line, sometimes obstacles or obstructions are encountered making the direct chaining impossible.

There may be following situations in the field:

1. Chaining is possible but ranging obstructed
2. Ranging possible but chaining obstructed
3. Both ranging and chaining obstructed.

Obstacles to ranging but not to chaining

When the ends of a chain line are not intervisible generally due to an intervening hill, ranging is not possible. Ranging in such cases has been discussed in Sec. 3.4 (Fig. 3.6) and Sec. 4.6 (Fig. 4.28 and Fig. 4.30).

Obstacles to chaining but not to ranging

A typical obstacle of this class is a water body such as pond, wider than a tape or chain length. There may be two possibilities as below:

1. It is possible to chain round the obstacle.
2. It is not possible to chain round the obstacle.

When it is possible to chain round the obstacle: There are several methods available to determine the distance AB which cannot be measured directly due to the obstacle but only a few are discussed below.

(a) Fig. 5.5a

Set out two equal perpendiculars AP and BQ. Then AB is equal to PQ.

(b) Fig. 5.5b

Erect a perpendicular AP at A, and measure AP and PB. Then AB is equal to $\sqrt{(PB)^2 - (AP)^2}$.

(c) Fig. 5.5c

Locate A such that $\angle APB$ is 90°, and measure AP and PB. Then AB is equal to $\sqrt{(AP)^2 + (PB)^2}$.

(d) Fig. 5.5d

Set out a straight line PQ at A, and measure AP, AQ, BP, and BQ. Then AB is computed as follows:

$$AB = \left[\frac{BP^2\,AQ + BQ^2\,AP}{PQ} - AP\,AQ\right]^{1/2}$$

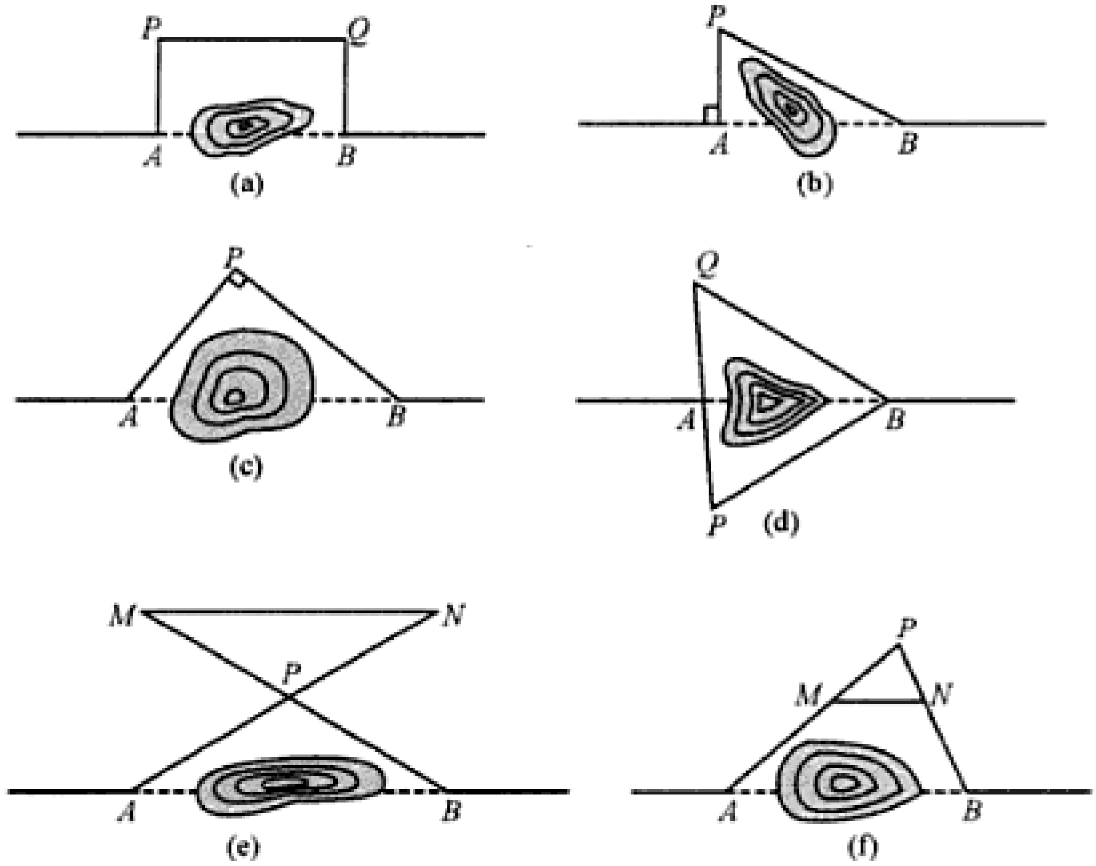


Fig. 5.5 Obstacle to chaining but not to ranging, and chaining possible round the obstacle

(e) Fig. 5.5e

Mark a point P clear of the obstacle, and a point N in line with A and P. Take $AP = PN$. Establish another point M by ranging so that it is in line with B and P. Take $BP = PM$. Measure MN which is equal to AB.

(f) Fig. 5.5f

Select a point P, and measure AP and BP. Mark M and N on the lines AP and BP, respectively, such that $PM = \dfrac{PA}{n}$ and $PN = \dfrac{PB}{n}$. Then $AB = nMN$.

In each of the above cases, the obstacle is surveyed taking offsets from the auxiliary lines.

When it is not possible to chain round the obstacle

A typical example of this case is a river. In such case, again there are several methods available to obtain the distance between two points across the river, but only a few shown in Fig. 5.6, are discussed below:

(a) Fig. 5.6a

Erect a perpendicular *AP* at *A*, and mark its midpoint *Q*. From *P* erect a perpendicular *PR* to *PA* such that *B*, *Q*, and *R* are in a straight line. Then $AB = PR$.

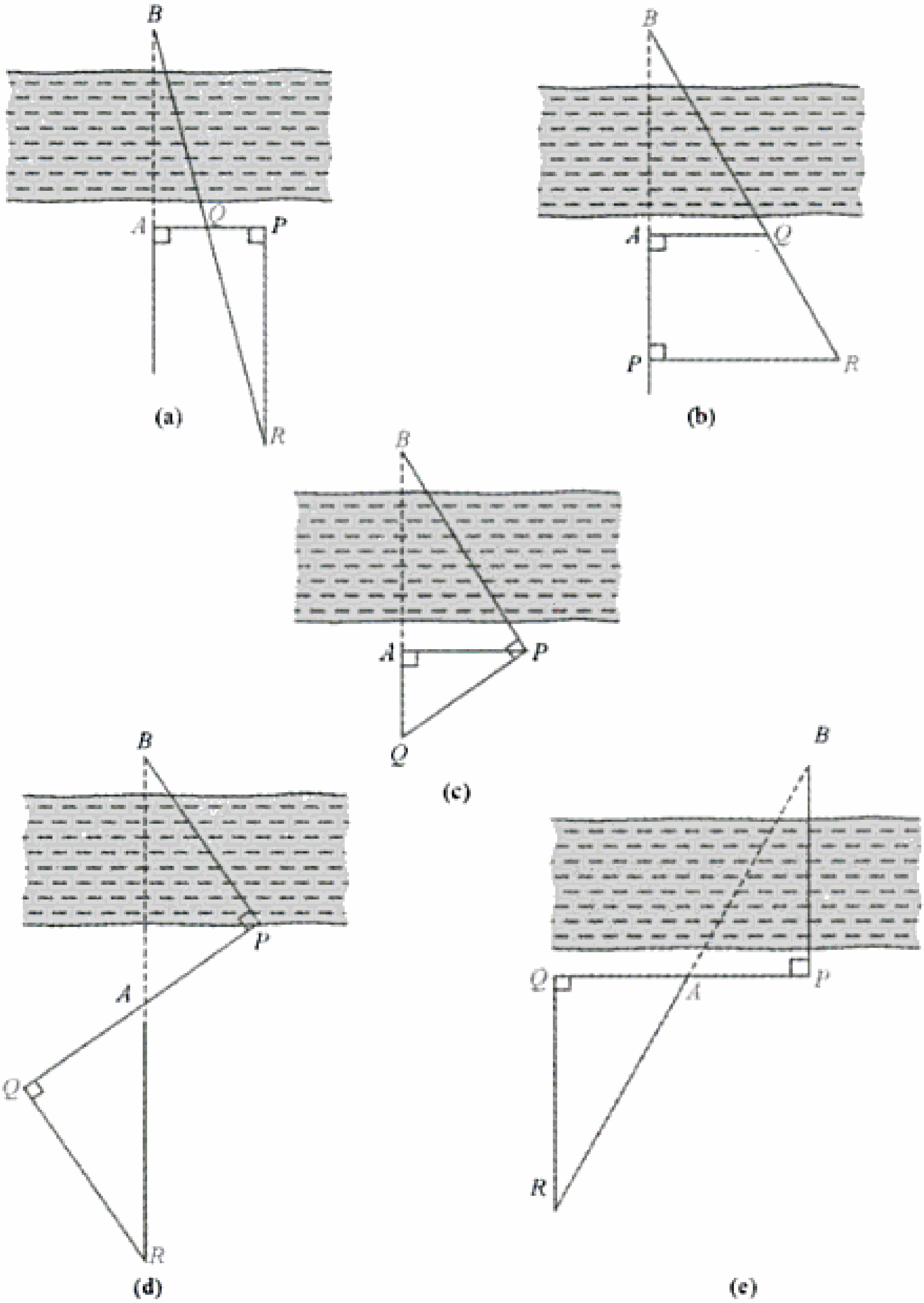


Fig. 5.6 Obstacle to chaining but not to ranging, and chaining not possible round the obstacle

(b) Fig. 5.6b

Mark a point P such that P, A, and B are in a straight line. Erect two perpendiculars AQ and PR at A and P, respectively, such that B, Q, and R are in a straight line. Measure AP, AQ, and PR. From two similar triangles ABQ and PBR, we have

$$\frac{AB}{PB} = \frac{AQ}{PR}$$

Taking $PB = AB + AP$, we get

$$AB = \frac{AP\,AQ}{(PR - AQ)}$$

(c) Fig. 5.6c

Mark Q such that Q, A, and B are in a straight line. Erect a perpendicular AP at A, and take a point P on the perpendicular such that angle QPB is a right angle. Measure AP and AQ. From two similar triangles BPQ and APQ, we have

$$\frac{AB}{AP} = \frac{AP}{AQ}$$

$$\text{Therefore,} \quad AB = \frac{AP^2}{AQ}$$

(d) Fig. 5.6d

Locate a point P such that angle BPA is a right angle. Mark a point Q such that it is in line with A and P, and AQ is equal to AP. Erect a perpendicular QR at Q taking R on the extension of the line BA. Then $AB = AR$.

(e) Fig. 5.6e

If the survey line crosses the river in a skewed direction, a line PAQ is set out approximately along the bank of the river. The points P and Q are located such that the angle APB is a right angle and AQ is equal to AP. A perpendicular QR is erected at Q such that B, A, and R are in line. Then $AB = AR$.

When both ranging and chaining are not possible

A building across the chain line is a typical example of this class of obstruction. Such problems require prolonging the line beyond the obstruction and determining the distance across it. The following are some of the methods of determining the distance across the obstacle.

(a) Fig. 5.7a

Choose two points A and C on one side of the obstacle, and erect perpendiculars AE and CG of equal lengths. Join E and G, and prolong EG past the obstacle. Select two points H and F on the prolongation of EG, and erect perpendiculars HD and FB equal to AE. Join D and B, and prolong DB. Then $CD = GH$.

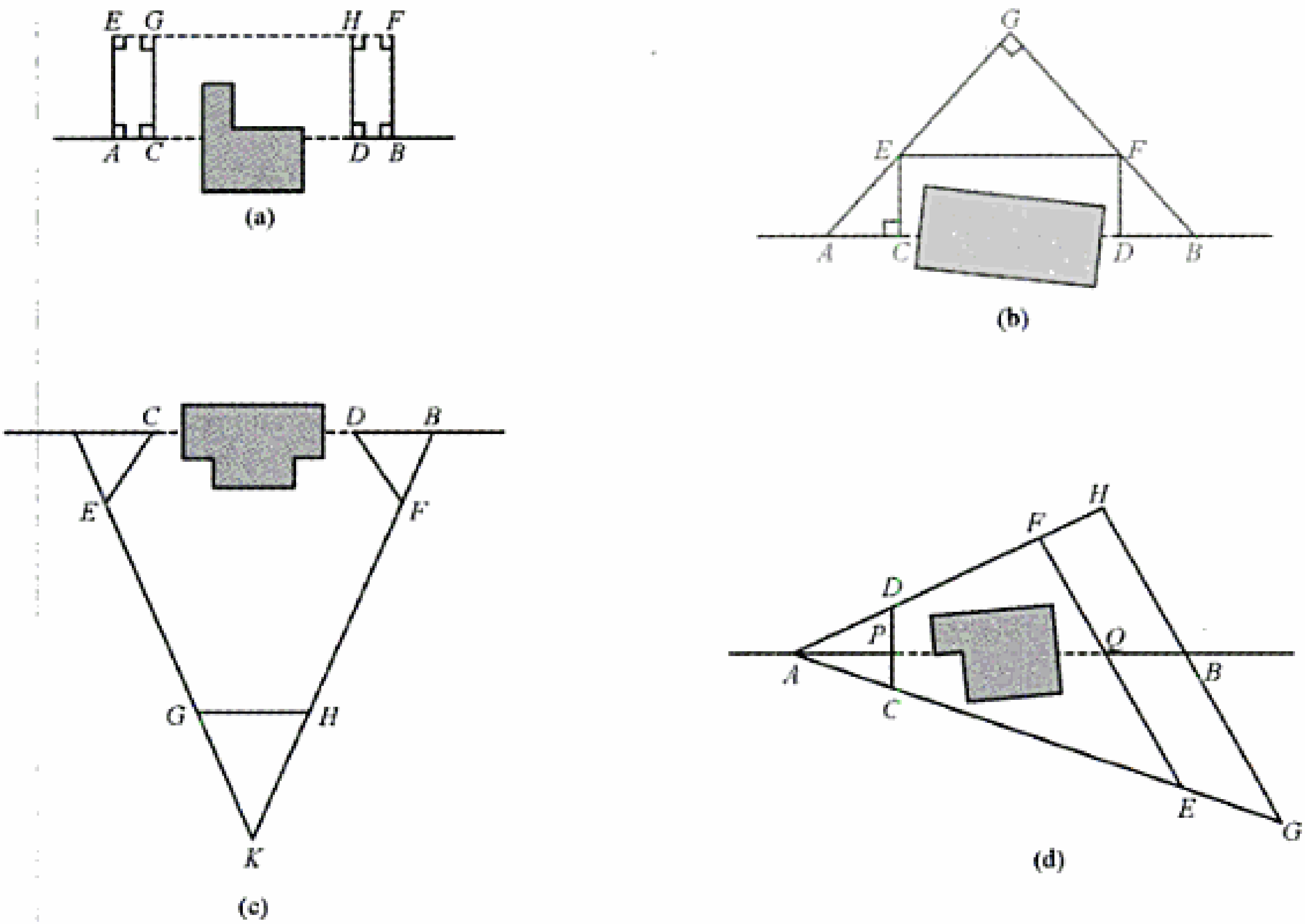


Fig. 5.7 Obstacle to both ranging and chaining

(b) Fig. 5.7b

Choose a point C, and erect a perpendicular CE of any convenient length on the chain line. Mark A such that AC equal to CE. Join A and E and prolong AE to any convenient point G. At G, set a right angle AGB such that $GB = GA$. Select a point F on GB taking $GF = GE$. From F and B, arcs of radii AC are swung, obtaining the intersection D. The line BD will be in range with the chain line, and CD will be equal to EF.

(c) Fig. 5.7c

Select the points A and C on the chain line, and construct an equilateral triangle ACE by swinging arcs. Join A and E, and prolong it to any point K. Select any point G on AK, and construct an equilateral triangle GKH. Join K and H, and produce KH to B such that $KB = KA$. Select F on KB, and construct an equilateral triangle BDF. The direction DB will be in range with the chain line.

The length CD will be equal to $AB - AC - DB$. As AB is equal to AK, $CD = AK - AC - DB$.

(d) Fig. 5.7d

Select the points A and P on the chain line and set a line CPD at any angle to AP. Join A and C, and produce AC to E such that $AE = n.AC$. Similarly produce AD to F such that $AF = n.AD$. Join E and F and select a point Q on EF such that $FQ = n.PD$. Produce AE and AF to G and H, respectively, such that $AG = n'.AC$ and $AH = n'.AD$. Join G and H and select B on GH such that $BH = n'.PD$. Join Q and B. The line QB will be in range with the chain line. The distance PQ is

$$= AQ - AP$$

But $AQ = nAP$

therefore $PQ = nAP - AP$

$= (n - 1)AP$

ILLUSTRATIVE EXAMPLES

Example 5.1

A chain line ABC crosses a river. The points B and C are situated on near and opposite banks of the river, respectively. The bearing of a line BD, perpendicular to AC at B, and 50 m long, is 45°20′. The measured bearing of DC is 264°50′. Determine the width BC of the river.

Solution (Fig. 5.8):

Bearing of DB = Bearing of BD + 180°

$= 45°20' + 180° = 225°20'$

$\angle CDB$ = Bearing of DC – bearing of DB

$= 264°50' - 225°20' = 39°30'$

From ΔBCD, we have

$$\tan CDB = \frac{BC}{BD}$$

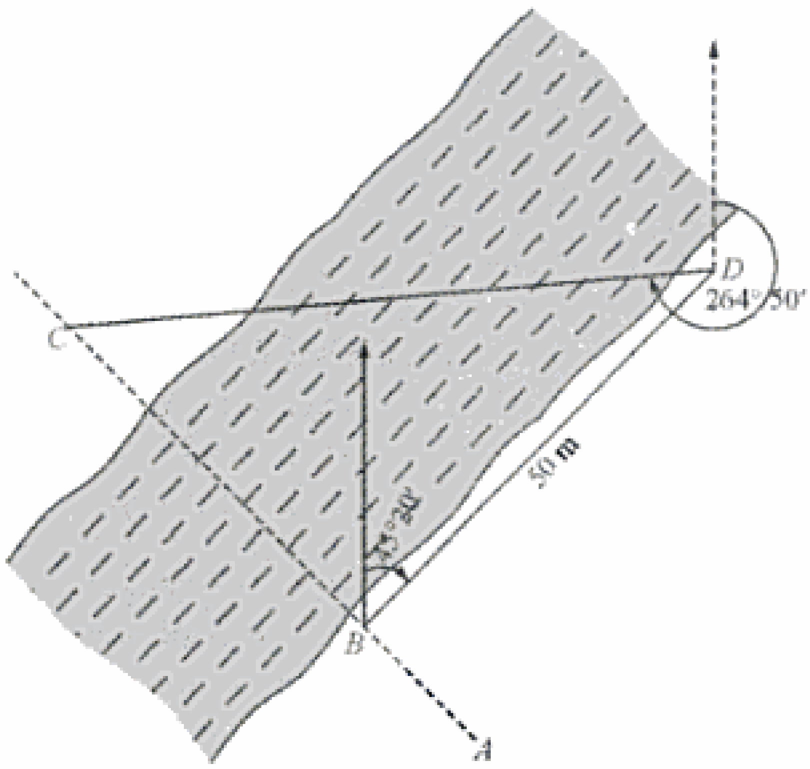


Fig. 5.8

or $BC = BD \tan CDB$

$= 50 \times \tan 39°30'$

$= \mathbf{41.22\ m.}$

Example 5.2

Two stations P and Q on the main survey line were taken on the opposite sides of a pond. On the right of PQ, a line PR, 210 m long was laid down and another line PS, 260 m long was laid down on the left of PQ. The points R, Q, and S are on the same straight line. The measured lengths of RQ and QS are 85 m and 75 m, respectively. What is the length of PQ?

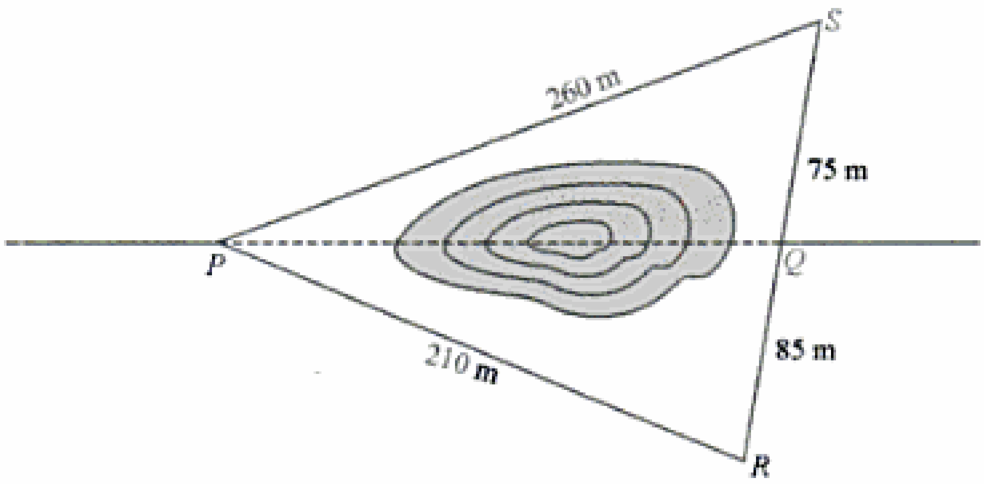


Fig. 5.9

Solution (Fig. 5.9):

It is given that

$RQ = 85$ m and $QS = 75$ m

$RS = RQ + QS$

$= 85 + 75 = 160$ m

Let $\angle PSR = \theta$

$$\text{Then } \cos\theta = \frac{SP^2 + SR^2 - PR^2}{2\,SP\,SR}$$

$$= \frac{260^2 + 160^2 - 210^2}{2 \times 260 \times 160}$$

$= 0.5901442$

Now considering ΔPQS, we have

$$\cos\theta = \frac{SP^2 + SQ^2 - PQ^2}{2\,SP\,SQ}$$

or $PQ = \sqrt{(SP^2 + SQ^2 - 2\ SP \times SQ\ \cos\theta)}$

$= \sqrt{(260^2 + 75^2 - 2 \times 260 \times 75 \times 0.5901442)}$

$= \mathbf{224.1\ m.}$

at the compass station. Brake pin may be used to dampen the oscillations of the needle by pressing it. The sun glasses provided at the eye vane may be used to sight the bright objects. When the instrument is not in use, the object vane frame may be folded on the glass lid. It automatically presses against a bent lever, which lifts the needle off the pivot and holds it against the glass lid.

Surveyor's compass

Surveyor's compass is very much similar in construction to the prismatic compass. It is different to the prismatic compass in the following respect:

1. The magnetic needle is not attached to the graduated ring and floats freely over the pivot.
2. The graduated ring is directly attached to the circular box.
3. No mirror is attached to the object vane for sighting the objects at higher elevation or depression.
4. The graduations on the ring are in quadrental system with 0° marks at north and south ends, 90° marks at east and west ends.
5. Readings are taken against the north end of the needle.

A surveyor's compass is shown in Fig. 5.14 and a comparison between a prismatic compass and a surveyor's compass is given in Table 5.1.

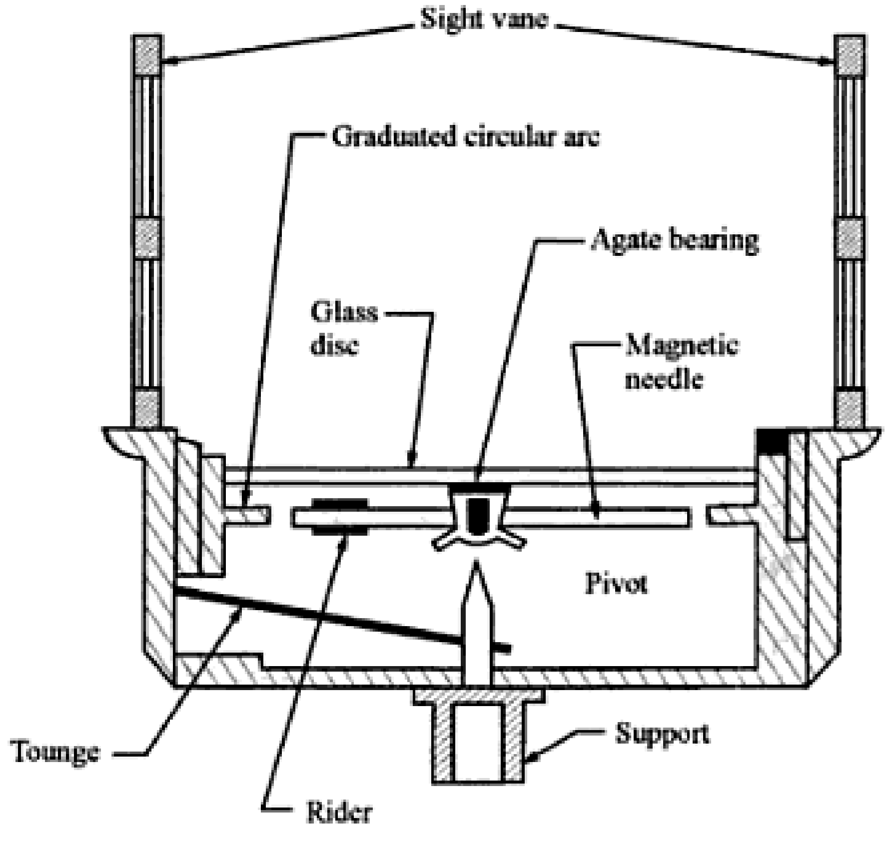


Fig. 5.14 Surveyor's compass

Trough and tubular compasses

Trough and tubular compasses are used as adjuncts to other instruments. They are not complete surveying instruments by themselves. They are used only to indicate magnetic meridian or to set the instrument in the magnetic meridian to measure the magnetic bearing of the lines.

A trough compass is generally used as an accessory to a plane table or theodolite, while a tubular compass is used only with a theodolite.

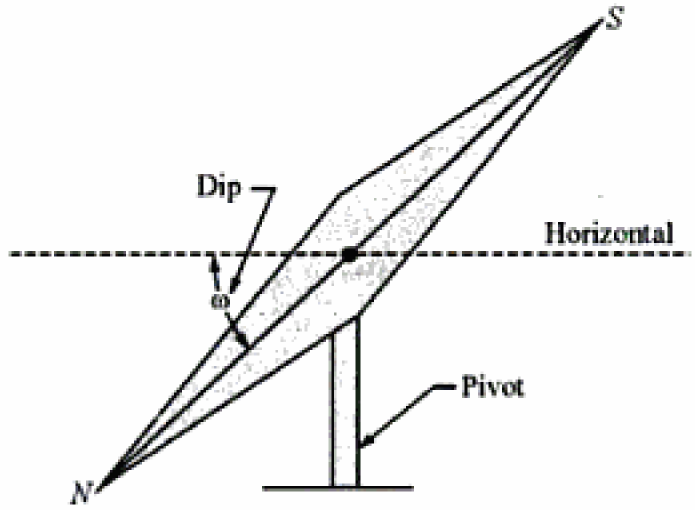


Fig. 5.15 Dip of a magnetic needle

To counteract the dip at any place to keep the magnetic needle in horizontal position, a sliding rider weight in the form of a brass or a silver wire coil, is provided towards the higher end of the needle at a suitable distance from the pivot.

5.5 MAGNETIC DECLINATION

The horizontal angle between the true or astronomic meridian and magnetic meridian is called the *magnetic declination.*

If the north end of the compass needle points to east of the true meridian as shown in Fig. 5.16, the declination is said to be east and if it points to the west of the true meridian the declination is said to be west.

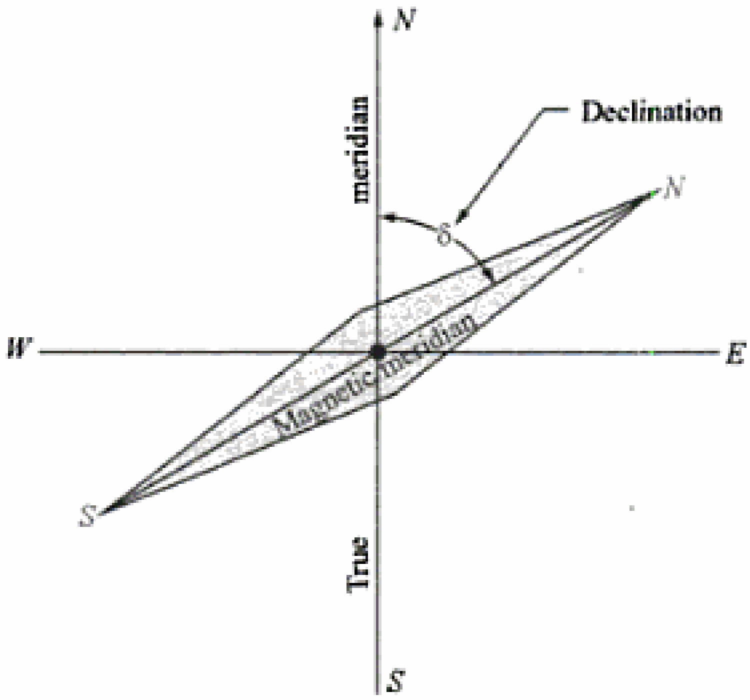


Fig. 5.16 Declination east

Isogonic Chart

The imaginary lines passing through points of equal magnetic declination are known as *isogonic lines* or *isogons*.

The lines passing through points where magnetic declinations are zero, are called *agonic lines*. Isogonic charts are published by agencies like U.S. National Ocean Survey and Survey of India. The charts also show the lines of equal annual rates of change of magnetic declination. The date on which the map was made and annual rates of change may also be given to help in prediction of the magnetic declination at any place on a particular day.

Variations in Magnetic Declination

The variations in magnetic declination at any place may be

1. Regular or periodic, and
2. Irregular.

Regular or periodic variations

These variations are further classified as:

(a) Secular (with periods of about 300 yrs.),
(b) Annual, and
(c) Dirunal or daily.

Secular or periodic variation

Like a pendulum, the magnetic meridian swings in one direction for perhaps 150 yrs until it gradually comes to rest and then swings in the other direction for 150 yrs. Thus the cycle is completed in about 300 years. The variation follows approximately a sine curve. The rate of change per year, however, varies irregularly. The causes of the secular variations are not well understood. To estimate the secular variation, observations are made at various places throughout the world.

The secular variations at a place are quite large and must be taken into account.

Annual variation

The annual variation is a yearly swing of small magnitude. It amounts to less than 01′ from the mean position. It is practically insignificant compared to the secular variation.

Dirunal or daily variation

The variation of the declination over the course of a day from the mean position is called dirunal variation. It is generally in the range of 3′ to 12′ at various places and is also called the Solar-dirunal variation. The value is considered to be well within the range of error expected in compass survey. The dirunal variation mainly depends upon the locality, season, time, and year.

Irregular variation

Magnetic disturbances or magnetic storms usually associated with sunspots cause irregular variation. Such variations are random and uncertain, thus, unpredictable. They may amount to a degree or more, particularly at high latitudes.

Effect of Variation

The variation in magnetic declination changes the direction of magnetic meridian at a place, thus changing the magnetic bearings of the survey lines. Old survey records in which the directions of survey lines were determined with a magnetic compass can only be retraced accurately if the declination of the place at the time of survey and the present declination are known.

The date of survey, magnetic declination, and annual change of the secular variation should be noted on the plan prepared by compass survey for applying corrections for declination in future.

ILLUSTRATIVE EXAMPLES

Example 5.5

The magnetic bearing of a line is 62°30′. What is the true bearing of the line if the magnetic declination is (*a*) 3°45′ W, (*b*) 4°10′ E?

Solution:

(a) See Fig. 5.17a

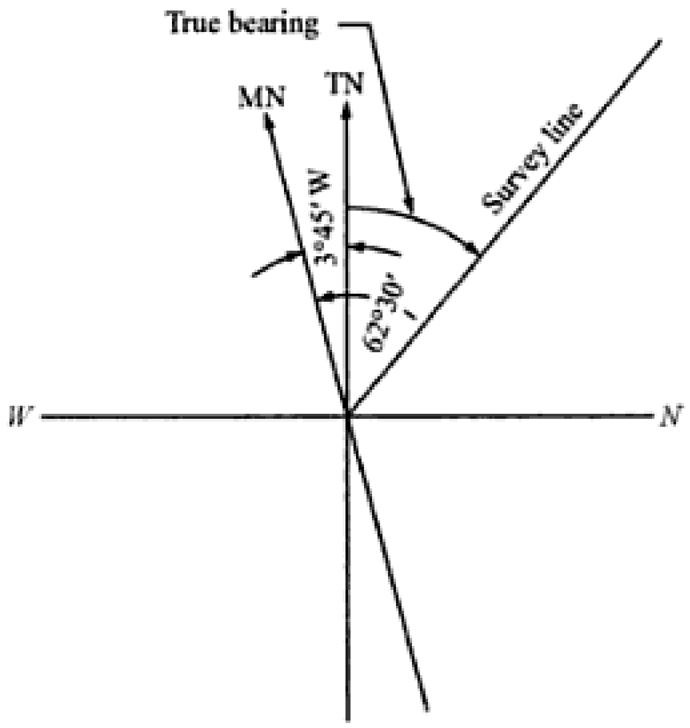
True bearing = magnetic bearing – declination
= 62°30′ – 3°45′
= **58°45′**.

(b) See Fig. 5.17b

True bearing = magnetic bearing + declination
= 62°30′ + 4°10′
= **66°40′**.

Example 5.6

The magnetic bearing of a line *PQ* is S 46°15′ E. Calculate the true bearing of *PQ* if the magnetic declination is (*a*) 4°45′ E, (*b*) 3°15′ W.



(a)

Fig. 5.17

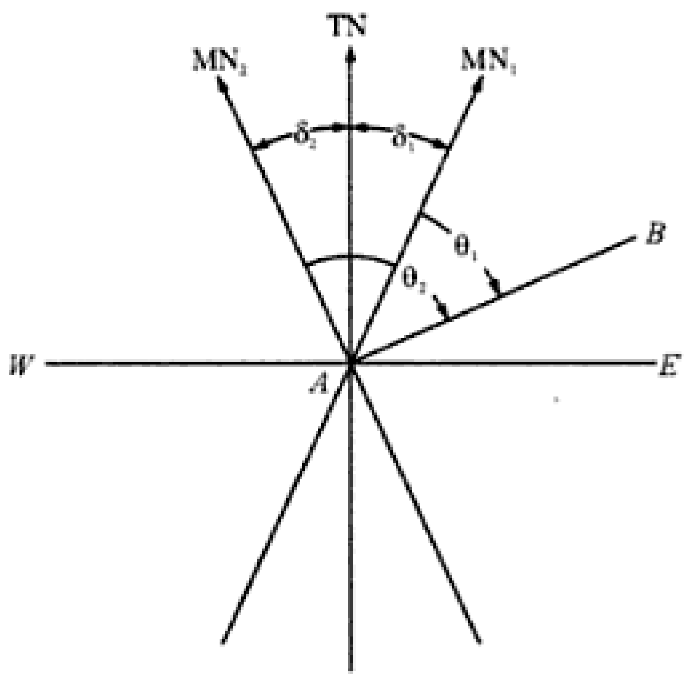


Fig. 5.19

Example 5.8

A line *AC* had the magnetic azimuth of 67°15′in 1960. If the magnetic declination was found to be 21°30′ E by interpolation from an isogonic chart dated 1950 with an annual change of 1′ westward. Determine the true azimuth of line *AC*.

Solution (Fig.5.20):

Magnetic azimuth in 1960	= 67°15′
Declination in 1950	= 21°30′ E
Change in declination from 1950 to 1960, i.e., for 10 years	= − 10′ @ 1′ per year westward
Magnetic declination in 1960	= (21°30′ − 10′) E
	= 21°20′E
True azimuth	= 67°15′ + 21°20′
	= **88°35′.**

Example 5.9

A magnetic bearing of N 43° 45′ W is recorded on an old plan of 1940. It is desired to relocate this direction on the ground in 1975. An isogonic chart of 1960 shows a declination of 12° 00′ W for the survey site, with an annual change of 3′ eastward. Determine the magnetic bearing that is to be used to reestablish the direction of the line.

Solution (Fig.5.21):

Magnetic declination in 1960	= 12°00′ W
Change in magnetic declination in 20 years @ 3′ per year eastward from 1940 to 1960	= 1°00′

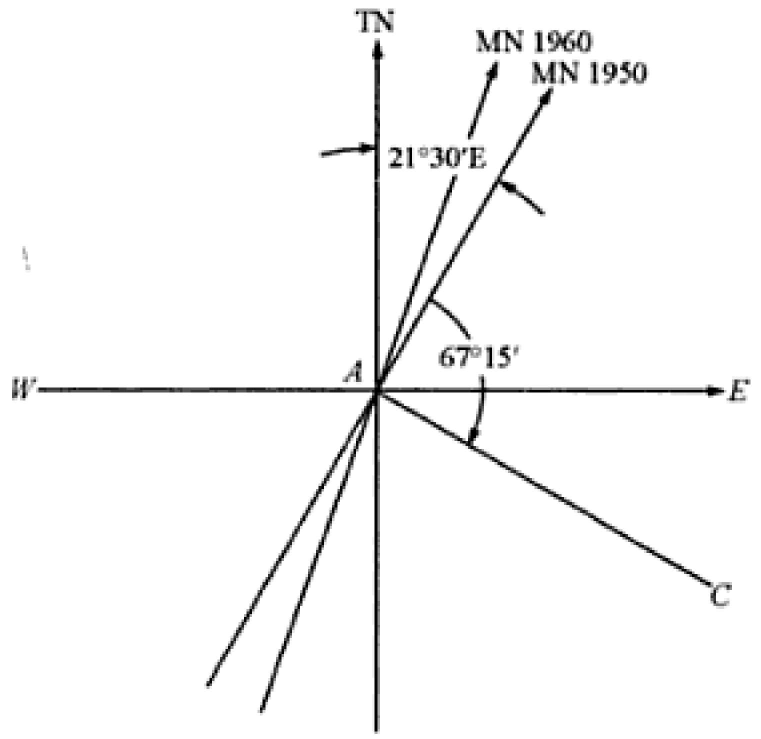


Fig. 5.20

Magnetic declination in 1940 = 12°00′ W + 1°00′
= 13°00′ W
Magnetic bearing in 1940 = N43°45′ W
Change in magnetic declination = 45′
in 15 years @ 3′ per year
eastward from 1960 to 1975

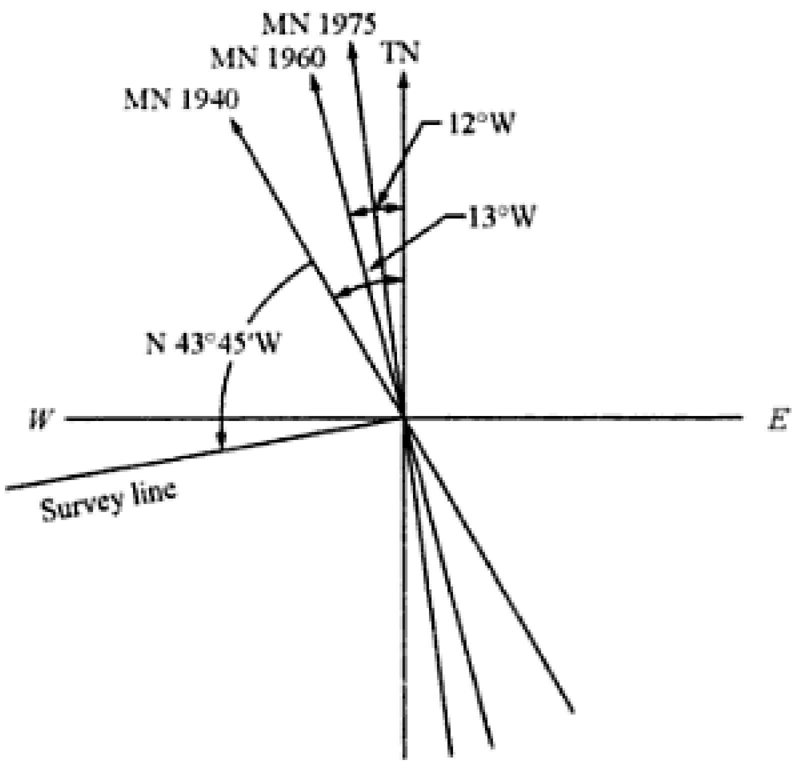


Fig. 5.21

Magnetic declination in 1975 $= 12°00' \text{ W} - 45'$
$= 11°15' \text{ W}$

Difference in magnetic declinations $= 13°00' \text{ W} - 11°15' \text{ W}$
from 1940 to 1975 $\Delta\delta = 1°45'$

Magnetic bearing in 1975 = Magnetic bearing in 1940 + $\Delta\delta$
$= \text{N}43°45' \text{ W} + 1°45'$
$= \textbf{N } \mathbf{45°30'} \textbf{ W}.$

Setting off Declination on the Compass

Set up the compass so that the north end of the magnetic needle reads 0° on the graduated ring. Now take the line of sight by setting off the known declination δ on the compass circle from the direction shown by the magnetic needle as shown in Fig. 5.22.

5.6 LOCAL ATTRACTION

The deviation of the magnetic needle from the magnetic meridian arising from local sources such as object of iron or steel, some kinds of iron ore, and currents of direct electricity, is called *local attraction* or *local disturbance*. Even small items made of iron or steel such as pen, belt buckle, wristwatch case, taping arrows, and steel tapes cause local attraction. In certain localities, particularly in towns, its effect is so pronounced as to render the magnetic needle of no value in determining directions.

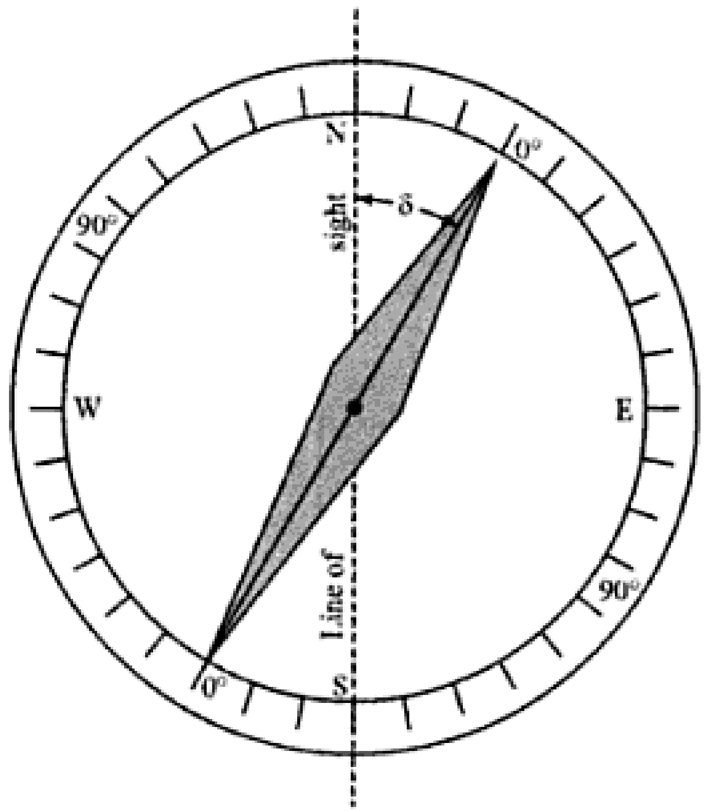


Fig. 5.22 Setting off declination on compass circle

Deviation of the magnetic needle from the magnetic meridian alters the magnetic bearing, and consequently the observed magnetic bearings will be in error due to local attraction. To detect local attraction it is essential to take both the fore bearing and back bearing of each line. If the bearings differ by 180°, there is no local attraction in the area provided there are no instrumental and other errors.

Correction for Local Attraction

For applying correction for local attraction it assumed that all the bearings measured at a particular station, are affected equally. The measured bearing of the traverse lines (*cf.*, Sec. 4.7 and Chapter 9) can be corrected for local attraction using either of the following methods.

Method-I: In this method, the line is found out whose fore bearing and back bearing differ exactly by 180°. For such lines, there is no local attraction at both the ends of the line and, therefore, all the bearings taken at these stations will be free from local attraction.

This helps in determining the corrections at the remaining traverse stations and then in determination of corrected bearings of the traverse lines. The method is explained in Example 5.10.

Method-II: As all the bearings taken at a station are equally affected, the included angle between two lines at this station obtained from the magnetic bearings of the two lines, will be free from local attraction.

In this method, the included angles between different lines of a traverse are found out from the observed bearings, including for the line which is not affected by local attraction. The corrected bearings of all other lines are determined from the computed included angles. The method is explained in Example 5.11.

ILLUSTRATIVE EXAMPLES

Example 5.10

A closed compass traverse was conducted round a forest and the following whole-circle bearings were observed. Determine which of the stations suffer from local attraction and compute the values of the corrected bearings.

Line	Fore bearing	Back bearing
AB	74°20′	256°00′
BC	107°20′	286°20′
CD	224°50′	44°50′
DA	306°40′	126°00′

Solution [by Method-I (Fig.5.23)]:

As the fore bearing and back bearing of *CD* differ exactly by 180°, stations *C* and *D* are free from local attraction. Hence, stations affected by local attraction are *A* and *B*. Further, all the bearings observed at *C* and *D* are also free from local attraction.

Therefore, the fore bearing 306°40′ of *DA* is correct.

Correct back bearing of *DA* = 306°40′ − 180° = **126°40′**

Error at A = 126°00′ − 126°40′ = − 40′

Correction at *A* = + 40′

Correct fore bearing of AB = 74°20′ + 40′ = **75°00′**
Correct back bearing of AB = 75°00′ + 180° = **255°00′**
Error at B = 256°00′ − 255°00′ = + 1°00′
Correction at B = − 1°00′
Correct fore bearing of BC = 107°20′ − 1°00′ = **106°20′**
Correct back bearing of BC = 106°20′ + 180° = **286°20′**.

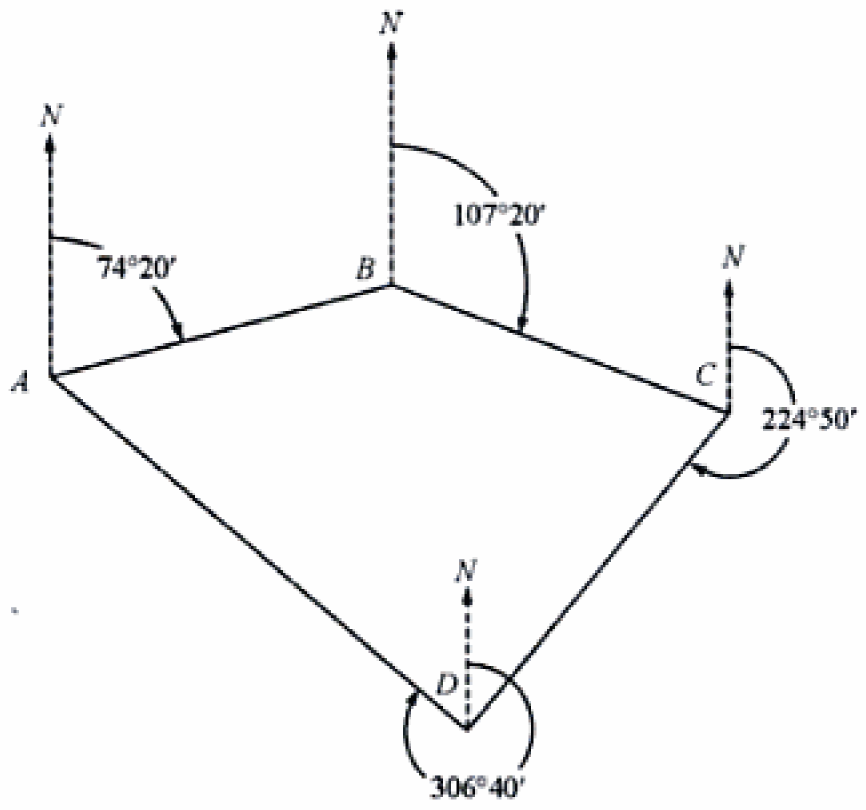


Fig. 5.23

Check: The computed back bearing of *BC* is same as the observed back bearing at C which is free from local attraction.

Line	Correct fore bearing	Correct back bearing
AB	75°00′	255°00′
BC	106°20′	286°20′
CD	224°50′	44°50′
DA	306°40′	126°40′

Example 5.11

Determine the corrected bearings of the traverse lines given in Example 5.10 by Method-II.

Solution [by Method-II (Fig. 5.23)]:
Interior angle at A = bearing of AD − bearing of AB
= 126°00′ − 74°20′ = 51°40′

Interior angle at B = bearing of BA – bearing of BC

= $256°00' - 107°20' = 148°40'$

Interior angle at C = bearing of CB – bearing of CD

= $286°20' - 224°50' = 61°30'$

Exterior angle at D = bearing of DA – bearing DC

= $306°40' - 44°50' = 261°50'$

Interior angle at D = $360° - 261°50' = 98°10'$

Sum of interior angles = $51°40' + 148°40' + 61°30' + 98°10'$

= $360°00'$

Theoretical sum = $(2n - 4) \times 90° = (2 \times 4 - 4) \times 90°$

= $360°$

where n is number of lines in a traverse

Therefore, no correction is required for the included angles. If the computed sum minus the theoretical sum is dA, which is the error, the correction to each included angle will be $-\dfrac{dA}{n}$. After applying the correction, calculate the corrected bearings as below.

As the fore bearing and back bearing of CD differ exactly by 180°, stations C and D are free from local attractions. Hence the observed fore bearing of CD, back bearing of BC, back bearing of CD and fore bearing of DA are the correct bearings.

Correct fore bearing of DA = $306°40'$ (given)

Correct back bearing of DA = $306°40' - 180° = 126°40'$

Correct fore bearing of AB = Correct back bearing of DA – included angle at A

= $126°40' - 51°40'$ = $\mathbf{75°00'}$

Correct back bearing of AB = $75°00' + 180°$ = $\mathbf{255°00'}$

Correct fore bearing of BC = Correct back bearing of AB – included angle at B

= $255°00' - 148°40'$ = $\mathbf{106°20'}$

Correct back bearing of BC = $106°20' + 180°$ = $\mathbf{286°20'}$.

Check: The computed back bearing of BC is same as the observed back bearing at C which is free from local attraction.

Line	Correct fore bearing	Correct back bearing
AB	75°00′	255°00′
BC	106°20′	286°20′
CD	224°50′	44°50′
DA	306°40′	126°40′

5.7 CHAIN AND COMPASS SURVEY

An area bounded by a traverse may be surveyed with chain and compass in the following steps:

1. Reconnaissance of the area
2. Measurement of the directions
3. Measurement of lengths and taking offsets.

Reconnaissance

The entire area to be surveyed is inspected to collect the following information:

1. The adjacent traverse stations are intervisible.
2. Taping between the traverse stations is not obstructed.
3. Survey lines are as close to the details as possible.
4. Survey lines are minimum in number and as long as possible.

Measurement of Directions

The directions of survey lines are measured with magnetic compass by free or loose-needle method (*cf.*, Sec. 9.5.4). The fore and back bearings of each line are measured independently by setting up compass at each successive station to avoid accumulation of errors. If error between fore and back bearings of a line exceeds the limit of permissible error, *i.e.*, 15′, the bearings of the line are observed again. Even if, on checking, the error remains, it may be due to local attraction at one or both the stations of the line. Before using the observed bearings in traverse plotting, they should be corrected for local attraction. The accuracy of field work mainly depends upon the accuracy with which bearings of survey lines are observed. Hence, to have less number of bearings, the survey lines of traverse legs should be as long as possible so that the number of lines are less.

A theodolite fitted with a magnetic compass is also used, to measure the directions. The following two methods are commonly used to measure the directions in compass surveying.

Loose-needle method

In this method, the direction of the magnetic meridian is established at each station of the traverse (*cf.*, Sec. 9.3), and the directions of the line are determined with reference to the magnetic meridian. As the magnetic bearings are read directly with a compass, the accuracy of the loose-needle method is that of the compass and not that of the theodolite. The loose-needle method is rarely used in a theodolite traverse.

Fast-needle method

In the fast-needle method, the magnetic meridian is established only at the starting station. The magnetic bearing of the first line is measured directly, and the magnetic bearings of the other lines are computed indirectly from the magnetic bearing of the first line and the included angles. Although the magnetic bearing of the first line has the accuracy of the compass, the difference of bearings of two adjacent lines has the accuracy of the theodolite. The method is more accurate than the loose-needle method, and is generally preferred.

Measurement of Length and Taking Offsets

The length of all the survey lines are measured by taping or chaining, and offsets are taken for the details on either side of the survey lines. The recording of the field notes is done in a field book as discussed in Sec. 5.2.3.

Plotting of Traverse Survey

Before plotting a traverse survey it should be ensured that all observed bearings are correct to have a perfect geometrical figure based on the field notes. To decide a proper layout of the plan on the drawing sheet, a rough sketch of the traverse should be drawn.

Plotting a compass traverse and its adjustment are given in Chapter 9.

5.8 ERRORS IN COMPASS SURVEY

The errors in compass survey may be classified as:

1. Instrumental errors
2. Observational or personal errors
3. Errors due to natural causes.

Instrumental Errors

These errors arise due to faulty adjustments and defective parts of the instruments, such as:

1. The needle not being perfectly straight
2. Pivot being bent
3. Sluggish needle
4. Blunt pivot points
5. Improper balancing weight
6. Sight vanes not being vertical
7. Line of sight not passing through the centre of the ring
8. Divisions of the graduated ring not being equal
9. Graduated ring not being horizontal.

Observational or Personal Errors

These errors include the following:

1. Inaccurate levelling of the compass box
2. Inaccurate centering
3. Inaccurate bisection of ranging rods
4. Inaccurate reading and recording.

Errors Due to Natural Causes

These errors may be due to the following reasons:

1. Presence of magnetic substances near or below the station of observation
2. Irregular variations due to magnetic storms, etc.

5.9 PRECAUTIONS IN COMPASS SURVEY

The following precautions should be taken during a compass survey to minimise the instrumental and observational errors:

1. Setting up and levelling of the compass should be done carefully.
2. If the needle is vibrating, use brake pin to stop the vibrations.
3. Avoid parallax in sighting by looking along the needle.
4. Always keep the needle off the pivot when compass is not in use to avoid unnecessary wear of the pivot.
5. Selected survey stations should be away from the sources causing local attraction.
6. Surveyor should not carry iron or steel articles with them at the time of making observations.
7. Object and eye vanes must be kept vertical at the time of reading.

PROBLEMS

5.1 Explain the principle of chain surveying. What are the limitations of chain surveying? Write the conditions in which it is suitable and unsuitable.

5.2 Define the following terms used in chain surveying and explain their use in chain surveying:
(*i*) Main survey station; (*ii*) Tie station; (*iii*) Main survey lines; (*iv*) Tie lines; (*v*) Base line; (*vi*) Check line.

5.3 What are the instruments used in chain surveying? How is the chain survey executed in the field?

5.4 What do you understand by reconnaissance? State its importance in chain surveying.

5.5 What is a well-conditioned triangle? Why is it necessary to use well-conditioned triangles?

5.6 What factors should be considered in deciding the stations for chain survey?

5.7 Describe how you would range and chain a line between two points not intervisible because of an intervening hillock.

5.8 How would you chain a survey line if there is obstacle of the following type:
(*a*) Chaining possible but ranging obstructed.
(*b*) Ranging possible but chaining obstructed.
(*c*) Both ranging and chaining obstructed.

5.9 Discuss various methods for determining the width of a river.

5.10 Discuss briefly 'Recording Field Notes' in chain surveying.

5.11 Discuss the steps for plotting a chain survey.

5.12 A river is flowing east-west, and a survey line *ABC* crosses the river perpendicular to the direction of flow. *B* is situated on the near bank and *C* on the far bank. A line *BD*, 60 m long, was laid at *B*, perpendicular to *AB*, and the bearing of *DC* and *DA* were measured and found to be 33°30′ and 131°20′, respectively. If *AB* is 42 m, calculate the length of *BC*.

5.13 A survey line *AB* if extended, intersects a pond at *P* and *Q*. Two lines *PC* and *PD* are laid on both sides of the pond and measured as 900 m and 1150 m respectively. *C*, *Q*, and *D* lie in a straight line. If the measured lengths of *CQ* and *DQ* are 580 m and 620 m, respectively, calculate the length of *PQ*.

5.14 Two points *A* and *B*, *A* being on the near bank and *B* on the far bank of a river, were located to measure the width of the river. Two perpendiculars *AQ* and *CP* were erected at *A* and *C*, *C* being on *BA* produced at a distance of 40 m from A so that P, Q, and B are colinear. If the measured lengths of CP and AQ are 135 m and 70 m, respectively, determine the width of the river.

5.15 For determining the width of a river, flowing east to west, two points P and Q are selected on the southern bank such that the distant PQ is equal to 60 m. Point P is westward. The bearings of a point A on the northern bank, are observed to be 40°00′ and 335°30′, respectively, from P and Q. Determine the width of the river.

5.16 Differentiate between chain survey and compass survey.

5.17 What are different types of compass? Discuss each briefly and write their uses.

5.18 Explain with the help of neat sketches the graduations of a prismatic compass and Surveyor's compass.

5.19 Give comparison between a prismatic compass and a Surveyor's compass in a tabular form.

5.20 What do you understand by dip of a magnetic needle and isoclinic lines? Explain the method of counteracting the dip.

5.21 What is magnetic declination? What are different types of variation in declination. Discuss also the importance of isogonic lines.

5.22 Write a short note on the term "declination".

5.23 What is local attraction? How is it detected and eliminated?

5.24 Explain the methods of determination of the correct bearing of lines of a traverse if some stations are effected by local attraction.

5.25 Write a short note on survey of an area using chain and compass.

5.26 Discuss the sources of errors in compass survey. What are the precautions which a surveyor should take in compass survey?

5.27 Following are the observed magnetic bearings of the traverse legs:

Line	*PQ*	*QR*	*RS*	*SP*
F.B	124°30′	68°15′	310°30′	200°15′
B.B	304°30′	246°00′	135°15′	17°45′

At what stations local attraction is suspected ? Determine the correct bearings of the traverse legs and also calculate the included angles.

5.28 The following fore and back bearings were observed in an open traverse:

Line	F.B.	B.B.
1 – 2	2°15′	182°15′
2 – 3	174°15′	354°00′
3 – 4	223°00′	42°45′
4 – 5	166°30′	346°45′

Which stations are affected by local attraction and how much? Determine the true bearings of the lines if the magnetic declination in the survey area is 2°10′E.

5.29 Determine the correct bearings of the lines of the traverse ABCDE from the following data taken from a compass survey :

Line	F.B.	B.B.
AB	N 55°00′E	S 54°00′W
BC	S 68°30′E	N 66°30′W
CD	S 24°00′W	N 24°00′E
DE	S 77°00′W	N 75°30′E
EA	N 64°00′W	S 63°30′E

5.30 A compass was set on the station *P*, and the bearing of *PQ* was observed as 312° 20′. Then the compass was shifted to station *Q* and the bearing of *QP* was observed as 132°20′.
Is there any local attraction at *P* or *Q* ? Give a precise answer supported by rational argument.